The Spatial Distribution of Wages and Employment:
Testing the Helpman-Hanson model for Germany

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ABSTRACT

Using German district data we estimate the structural parameters of a new economic geography model as developed by Helpman (1998) and Hanson (1998). The advantage of the Helpman-Hanson model is that it incorporates the fact that agglomeration of economic activity increases the prices of local (non-tradable) services, like housing. This model thereby provides an intuitively appealing spreading force that allows for less extreme agglomeration patterns than predicted by the bulk of new economic geography models. Generalizing the Helpman-Hanson model, we do not only test for the spatial distribution of wages but also for employment and land prices. In general we not only find support for a spatial wage and employment structure but also for relevance of the key structural parameters of the theoretical model. A comparison of our model with two alternative explanations indicates that our model specification with land prices is to be preferred.

JEL-code: R10, R12, R23

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1. Introduction

Initiated by Krugman (1991) there has been a renewed interest in mainstream economics in recent years for the question how the spatial distribution of economic activity comes about. The literature on the so called new economic geography or geographical economics, shows how modern trade and growth theory can be used to give a sound theoretical foundation for the location of economic activity across space.\(^2\) The seminal book by Fujita, Krugman and Venables (1999) develops and summarizes the main elements of the new economic geography approach. The emphasis in this book is strongly on theory and empirical research into the new economic geography is hardly discussed at all. As already observed by Krugman (1998, p. 172), this is no coincidence since there is still a lack of direct testing of the new economic geography models or its implications. In a review of Fujita, Krugman and Venables (1999), Neary (2001) concludes that empirical research is lagging behind. In order to make progress an empirical validation of the main theoretical insights is called for. The reason that the empirical research lags behind is that the new economic geography models are characterized by non-linearities and multiple equilibria which makes empirical validation relatively difficult.

There is a substantial amount of empirical research that shows that location matters, but there are indeed still relatively few attempts to specifically test for the relevance of the structural parameters of new economic geography models (see the survey by Overman, Redding and Venables (2001)). A notable exception is the work by Gordon Hanson (1998, 1999). Hanson uses a new economic geography model developed by Helpman (1998) and then directly tests for the significance of the model parameters. Based on US county-data, he finds confirmation for his version of the Helpman model but that the underlying demand linkages are of limited geographical scope. In this paper we apply the Helpman-Hanson model to the case of Germany. The goal of the paper is threefold.

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\(^2\) Elsewhere, see in particular Brakman, Garretsen and van Marrewijk (2001), we have argued that is more accurate to use the phrase “geographical economics” instead of “new economic geography” because the approach basically aims at getting more geography into economics rather than the other way around, but we stick here to the latter to avoid confusion.
First, we establish whether the Helpman-Hanson model can be verified, that is to say we want to know whether the key model parameters are significant and offer a meaningful description of the basic spatial features of an economy, in our case the German economy. The main equation to be estimated will be a wage equation, central to this equation is the idea that wages will be higher in those regions that have easy access to economic centers because for those regions demand linkages are relatively strong.

Second, we extend the analysis by Hanson in two ways: we drop the assumption of real wage equalization as this is not very realistic for most countries but especially for re-unified Germany. We also allow for wage inflexibility since, as opposed to the USA, European countries like Germany have limited wage flexibility due to centralized wage setting. This requires alternative specifications to be estimated and we also use data that are more in line with the underlying model, in particular with respect to the non-tradable good (housing) that plays a key role in the model. The main geographical unit of analysis is the German city-district (*kreisfreie Stadt*).

Third, we compare our new economic geography model with two alternatives by also estimating a simple market potential function and a (neo-classical) wage curve. The former encompasses a wide range of theoretical approaches that somehow explicitly included a role for location or distance and as such represents a summary of alternative explanations, see Harrigan, 2001.

Even though the case of post-reunification Germany is thought to be well-suited for a new economic geography approach (see Brakman and Garretsen (1993) for an early qualitative attempt), the goal of the present paper is *not* to analyze whether our new economic geography model is the “best” model to analyze Germany after the fall of the Berlin Wall in 1989. We simply want to assess the empirical relevance of a particular new economic geography model, here for the case of Germany. The fall of the Berlin Wall creates a unique testing ground. A main problem in testing new economic geography models is whether or not the economy is in a long-run equilibrium and if so in which equilibrium. This problem is
reduced for the German case. We can assume that a few years into the reunification process the spatial allocation of economic activity is not an equilibrium and this helps us to choose between alternative specifications. In our paper we take the approach recommended by Hanson (2000) as a starting point. He concludes his survey of the empirical literature of spatial agglomeration by stating that the well-documented correlation of regional demand linkages with higher wages “would benefit from exploiting the well-specified structural relationships identified by theory as a basis for empirical work” (Hanson, 2000, p. 28).

The paper is organized as follows. In section 2 we visualize the idea that geography matters in Germany by showing a few maps. We also give a first indication of the importance of the housing prices, the most distinguishing variable of the Helpman-Hanson model. In section 3 we focus on the derivation of the empirical specification of the wage equation, discuss our data set and some estimation issues. Section 4 gives the main estimation results for the basic wage equation. In section 5 we address the implications of dropping the assumptions of real wage equalization and wage flexibility. Section 6 evaluates the value added of the new economic geography model by estimating an alternative model, the market potential function. Section 7 concludes the paper. In general we not only find support for a spatial wage structure but also for relevance of the key structural parameters of the theoretical model. A comparison of our model with two alternative explanations of a spatial wage structure indicates that our model specification with land prices is to be preferred.

2. Geography and Germany

In this section we briefly present some data in order to illustrate the spatial distribution of some key variables across Germany. A quick look at the Maps 1-3 below immediately shows that there are indeed geographical or spatial differences within Germany with respect to the economic variables that are at the heart of our model. Take, for instance, a look at Map 1 which gives GDP per km² for the German states (Bundesländer). This map shows that geographical differences with respect to GDP are quite large (and even more skewed than GDP per capita, not shown here). The map also indicates that GDP per km² is higher in

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1 For a similar attempt see Roos (2001)
the former West Germany and this is not only true for smaller city-states like Bremen, Hamburg and West-Berlin.\(^4\)

**Map 1.**

GDP per km\(^2\) in millions of DM (1994)

![GDP per km\(^2\) in millions of DM (1994)](image)

Source: Federal Statistical Office, Wiesbaden

Central in this paper is the Helpman-Hanson model. The key equation in this model, as will be explained in the next section, describes the spatial nature of (nominal) wages. Map 2 indicates that not only hourly (manufacturing) wages differ remarkably between states, but the map also suggests that in the eastern part of Germany wages are on average lower than in western Germany. The dividing line between high and low wages to some extent identifies the former border between East and West Germany. In the Helpman-Hanson model wages in a region are higher if that region is part of or close to a large market, proxied by GDP.

\(^4\) Maps 1-3 are based on information of 441 districts (\textit{Kreise}), which we aggregated to 16 states (\textit{Bundesländer}), to avoid information overload in the maps. The solid lines indicate the states, the dashed lines the districts.
This is in line with Maps 1 and 2 because these two maps suggest a positive correlation between nominal wages and GDP (per km²).

Map 2.

![Map of Germany showing average hourly wage in the manufacturing sector (1995)](image)

Source: Federal Statistical Office, Wiesbaden

Agglomeration in new economic geography models is, as geographers have known for a long time, the result of the combination of agglomerating and spreading forces. An important agglomerating force is for instance the size of the market (see Map 2). Among the spreading forces are the demand from immobile workers in peripheral regions, but also negative feedbacks in the core-regions such as congestion or the relatively high cost of housing and other local goods. An indication for the presence of these spreading forces in core regions are, for example, land prices. As Map 3 indicates, land prices in eastern Germany seem on average lower than in western Germany, but a possible dividing line between eastern and western Germany is less clear-cut than with respect to regional wages. Land prices can be looked upon as a proxy for housing prices and as will become clear in Section 3 housing...
prices are the spreading force in the Helpman-Hanson model. Hence, Maps 1-3 give a first indication of the spatial distribution of the three key variables in the theoretical model, nonimal wages, the size of the market (GDP) and housing prices (here proxied by land prices). Taken together these maps suggest that there is no random distribution of economic activity across Germany and that high wages go along with high gdp and high land prices. A look at the Kreise data on which the Maps are based confirms this conclusion. The highest (lowest) values for the three variables are invariably observed in western (eastern) German Kreise.

Map 3.

Prices of land per m² (1995)

Source: Federal Statistical Office, Wiesbaden

The inclusion of housing as a non-tradable consumption good lies at the very heart of the Helpman (1998) model and to illustrate (nothing more, but certainly nothing less) that
housing prices may indeed display a spatial structure, as suggested by Map 3, we estimate a market potential function, equation (1).

\[
\log (LP_r) = \kappa_1 \log [\sum_s Y_s e^{-\kappa_2 D_{rs}}] + \kappa_3 D_{east} + \kappa_4 D_{country} + \text{constant}
\]

where \(LP\) = land prices per m\(^2\) in city-district \(r\) in 1995, \(Y\) = GDP in city-district \(r\) in 1994, \(D_{rs}\) = distance between city-districts \(r\) and \(s\), \(D_{east}\) and \(D_{country}\) are two dummies for eastern-German and country districts respectively. \(\kappa_1 - \kappa_4\) are coefficients to be estimated, for more details on the data see section 3.2.

The estimation results, see Table 1, show that there is a “spatial land price” structure (\(\kappa_1 > 0\) and \(\kappa_2 > 0\)) which means that land prices in district \(r\) are higher if this district is near to districts with a high GDP. This is precisely what drives the Helpman-Hanson model. Also in line with other German evidence (see Sinn, 2000) is that land prices are significantly lower in country districts but notably also in East German districts.

### Table 1 Spatial land price structure

<table>
<thead>
<tr>
<th>(\kappa)</th>
<th>Coefficient</th>
<th>(t)-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\kappa_1)</td>
<td>0.408</td>
<td>(6.8)</td>
</tr>
<tr>
<td>(\kappa_2)</td>
<td>0.063</td>
<td>(3.8)</td>
</tr>
<tr>
<td>(\kappa_3)</td>
<td>-0.586</td>
<td>(-4.1)</td>
</tr>
<tr>
<td>(\kappa_4)</td>
<td>-1.361</td>
<td>(-10.5)</td>
</tr>
<tr>
<td>Adjusted (R^2)</td>
<td>0.726</td>
<td></td>
</tr>
</tbody>
</table>

\# obs.=151, estimation method: weighted least squares (WLS), result for constant not reported, \(t\)-statistic between brackets.

### 3. The Model and Data

#### 3.1 The Helpman-Hanson model
The benchmark model of the new economic geography, developed by Krugman (1991), is in general not suited for empirical validation. In the long run it produces for an intermediate range of trade costs only one or at most a very few (equally sized) locations with manufacturing economic activity. This is clearly not in accordance with the facts about the spatial distribution of manufacturing activity for the US or any other industrialized country because in reality we observe the co-existence of large and small locations. Furthermore, it lacks some of the spatial characteristics of agglomerations that have been found to be very relevant empirically, most importantly the tendency of prices of local (non-tradable) goods to be higher in agglomerations (see for example the survey by Anas, et al., 1998, and our Map 3 for that matter).

Given the observation that complete agglomeration is not in accordance with the facts, there are two alternatives. First, there are the new economic geography models based on forward and backward linkages with no interregional labor mobility (Krugman and Venables, 1995, 1996, and Venables, 1996). However, direct testing of these models is rather cumbersome because it requires detailed information about input-output linkages between firms on a regional level (see Combes and Lafourcade (2001) for an empirical test of this class of models). Second, there is the Helpman-Hanson model which combines the “best of the two worlds”. It shares with Krugman (1991) the emphasis on demand linkages (which are more easy to test for than input-output linkages) and at the same time, through the inclusion of a non-tradable consumption good (i.e housing), is capable of producing similar equilibria as the models based on input-output linkages and immobile factors of production. The price of housing in the Helpman (1998) model which increases with agglomeration, serves as an analogous spreading force as the rising wages in for instance Puga (1999). In fact, it can be shown that in terms of equilibrium outcomes the Helpman model yields similar results as the model in Krugman and Venables (1996) where there is no interregional labor mobility and the possibility of agglomeration arises through intricate input-output linkages between firms.

5 For the differences between Krugman (1991) and Helpman (1998), see Helpman (1998, pp. 49-53). For a very useful general framework to understand the different implications of models with and without interregional labor mobility see Puga (1999, 2001). For the observation that the Helpman model is
We briefly discuss the theoretical approach in Hanson (1998, 1999) and focus on the equilibrium conditions because these are needed to arrive at the basic wage equation that will be estimated. With one notable exception (the inclusion of a non-tradable good (housing)) the micro-foundation for the behavior of the individual consumers and producers is the same as in the seminal Krugman (1991) model. Here we only discuss the resulting equilibrium conditions and for the full model specification we refer to Hanson (1998, 1999).

In the model consumers derive utility from consuming a manufacturing good, which is tradable albeit at a cost, and from housing which is a non-tradable good between regions. The manufacturing good consists of many varieties and each firm offers one variety and this is modeled with well-known Dixit-Stiglitz formulation of monopolistic competition. The only factor input in the model is labor and labor is needed to produce the manufacturing good and labor can move between regions in the long run. In this set-up of the model the perfectly competitive housing sector serves as the spreading force, because housing (a non-tradable good) is relatively more expensive in the centers of production where demand for housing is high. As we will see below, apart from the inclusion of a homogenous non-tradable good (housing) at the expense of a homogenous tradable good (agriculture), there are no fundamental differences between Krugman (1991) and Helpman (1998). In particular in both models agglomeration is driven by demand linkages and the interregional mobility of labor.

This extension of core model thus allows for a richer menu of equilibrium spatial distributions of economic activity then the core model. As trade or transportation costs fall agglomeration remains a possible outcome but now also (renewed) spreading and partial agglomeration are feasible. Partial agglomeration means that all regions have at least some industry. Notwithstanding the different implications of Helpman (1998) compared to Krugman (1991) the equilibrium conditions (five in total) are very similar to the core model, in particular the equilibrium wage equation, which is central to the empirical analysis, is identical to the (normalized) equilibrium wage equation in Krugman (1991):

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\[
W_r = \left[ \sum_s Y_s I_s^{1-\varepsilon} T_s^{D_{rs} (1-\varepsilon)} \right]^{\varepsilon}
\]

\[
I_r = \left[ \sum_s \lambda_s \left( T_s^{D_{rs}} \right)^{1-\varepsilon} W_s^{1-\varepsilon} \right]^{1/(1-\varepsilon)}
\]

\[
Y_r = \lambda_r LW_r
\]

In which in equation (2) \(W_r\) is the region’s \(r\) (nominal) wage rate, \(Y\) is income, \(I\) is the price index for manufactured goods, \(\varepsilon\) is the elasticity of substitution for manufactured goods, \(T\) is the transport cost parameter, and \(T_{rs} = T^{D_{rs}}\), where \(D_{rs}\) is the distance between locations \(r\) and \(s\). Transport costs \(T\) are defined as the number of manufactured goods that have to be shipped in order to ensure that one unit arrives over one unit of distance. Given the elasticity of substitution \(\varepsilon\), it can directly be seen from equation (2) that for every region wages are higher when demand in surrounding markets \(Y_s\) is higher (including its own market), when access to those markets is better (lower transport costs \(T\)). Also regional wages are higher when there is less competition for the varieties the region wants to sell in those markets (this is the extent of competition effect, measured by the price index \(I\)).

Equation (3) gives the equilibrium price index for region \(r\), where this price index is higher if a region has to import a relatively larger part of its manufactured goods from more distant regions. Note that the price index \(I\) depends on the wages \(W\). Equation (4) simply states income in region \(r\), \(Y_r\), has to equal the labor income earned in that region, where \(\lambda_r\) is region \(r\)’s share of the total manufacturing labor force \(L\). \(^7\)

The main aim of our empirical research is to find out whether or not a spatial wage structure, that is a spatial distribution of wages in line with equation (2), exists for Germany. Equation (2) cannot be directly estimated as there are typically no time series of local price indices for manufactures (where local refers to the US county level in Hanson’s study and to the city-district level in our case). And, even more problematic (see equation (3)), the price index \(I\) is

\(^6\) For an in-depth analysis of the Krugman (1991) model see Fujita, Krugman and Venables (1999, chapters 4 and 5) or Brakman, Garretsen and van Marrewijk (2001, chapters 3 and 4). For the full specification of the Helpman-Hanson model, see Hanson (1998, pp. 9-12).

\(^7\) Note that the housing stock is owned by absentee landlords, that is no income is gained from owning houses. Dropping this assumption (by endowing each worker with a share of the housing stock) has no impact on the empirical specifications below (equation (7)), compare Hanson (1998) with Hanson (1999).
endogenous, and inter alia depends on each of the local wage rates, which makes a reduced form of equations (2) and (3) extremely lengthy and complex. These problems have somehow to be solved in order to estimate a spatial wage structure for Germany.

Hanson uses the following estimation strategy based on the remaining two equilibrium conditions. In order to arrive at a wage equation that can actually be estimated he rewrites the price index $I$ in exogenous variables which can actually be observed for his sample of US counties.

First, he uses:

\[ P_r H_r = (1 - \delta) Y_r \]  

Equation (5) states that the value of the fixed stock of housing equals the share of income spent on housing, where $P_r$ is the price of housing in region $r$, $H_r$ is the fixed stock of housing in region $r$ and $(1-d)$ is the share of income spent on housing and $d$ is thus the share of income spent on manufactures.  

Second, real wage equalization between regions is assumed:

\[ \frac{W_r}{p_r^{1-\delta} I_r^{\delta}} = \frac{W_s}{p_s^{1-\delta} I_s^{\delta}} \]  

Equation (6) is quite important. In fact, it is assumed that the economy has reached a long-run equilibrium in which real wages are identical. This implies that labor has no incentive to migrate (interregional labor mobility is solely a function of interregional real wage differences).  

The assumption of interregional labor mobility and the notion that agglomeration leads to interregional wage differences are not undisputed for a country like Germany with an allegedly “rigid” labor market, see in particular Puga (2001, p. 18) for implications of low labor mobility and no interregional wage differences from a new economic geography perspective. We return to this issue in section 5.

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8 Note that direct observation of a housing price index would imply that one only has to use equation (6) and that the use of equation (5) is no longer necessary. We will return to this in section 4.

9 Overman, Redding and Venables (2001, p. 17) discuss how the model used by Hanson can be seen as a specific version of a more general new economic geography model.
The importance of a non-tradable housing sector as a spreading force is implied by (6). A higher income $Y_r$ implies, ceteris paribus, higher wages in region $r$, see equation (2), but it also, given the stock of housing, puts an upward pressure on housing prices $P_r$, equation (5). Combining (5) and (6) allows us to rewrite the price index in terms of the housing stock, income and nominal wages. The equilibrium condition for the housing market can be written as $P_r = (1-\delta)Y_r/H_r$ and this expression for $P_r$ is then substituted into equation (6) and this gives the price index $I_r$ in terms of $W_r$, $Y_r$ and $H_r$. Substituting this in (1) results in a wage equation which can be estimated. This will also be the benchmark wage equation in our empirical analysis.

\[(7) \quad \log(W_r) = k_0 + \varepsilon^{-1} \log \left( \sum_s Y_{rs}^{\varepsilon+(1-\varepsilon)/\delta} H_{rs}^{(1-\varepsilon)/\delta} W_{rs}^{(\varepsilon-1)/\delta} T_{rs} \right) + \text{err}_r \]

Where $k_0$ is a parameter and $\text{err}_r$ is the error term. Equation (7) includes the three central structural parameters of the model, namely share of income spent on manufactures, $\delta$, the substitution elasticity, $\varepsilon$ and the transport costs, $T$. Given the availability of data on wages, income, the housing stock, and a proxy for distance, equation (7) can be estimated. The dependent variable is the wage rate measured at the US county level and Hanson finds strong confirmation for underlying model to the extent that the three structural parameters are significant and have the expected sign which, in terms of equation (7), means that there is a spatial wage structure. In section 4 we will begin our empirical inquiry of the German case by estimating equation (7) for our sample of German city districts.

3.2 Data and estimation issues

Before we turn to the estimation results a few words on the construction of our data set are in order. Germany is administratively divided into about 441 districts (Kreise). Of these districts a total of 119 districts are so called city-districts (kreisfreie Stadt), in which the district corresponds with a city. 114 of these city districts are included in the sample. We use district statistics provided by the regional statistical offices in Germany. The data set contains local variables, like the value added of all sectors in that district (GDP), the wage bill and the number of hours of labor in firms with 20 or more employees in the mining and manufacturing sector. Combining the latter two variables gives the regional wage $W_r$, which is measured as the average hourly wage in the manufacturing and mining sector. Since we
also want to analyze the cities’ *Hinterland* we also included 37 aggregated (country) districts, constructed from a larger sample of 322 country districts.\(^\text{10}\) The total number of districts in our sample is thus 151, namely 114 city districts and 37 country districts. Transport costs are, of course, a crucial variable. We do not use the geodesic distance between districts, because this measure does not distinguish between highways and secondary roads. Instead, distance is measured by the average car travel time (in minutes) between city district A to city district B. The data are obtained from the Route Planner 2000 (Europe, And Publishers, Rotterdam). For the data on the housing stock \(H\), required to estimate equation (7), we use the number of rooms in residential dwellings per district. In some of our estimations we also include one or more of the following regional variables, employment, income (personal income tax base) and land prices (*Baulandpreise*).

Since we only have one observation for each variable per district for the average hourly wage and for GDP (for 1995 and 1994 respectively) we have to estimate the wage equation (7) in levels and we therefore restrict ourselves to cross-section estimations. The estimation of an equation like equation (7) raises several estimation issues. First of all, there is the issue of the endogeneity of particular right hand side variables like \(Y_s\). In our case this problem is somewhat reduced by the fact that wage data are for 1995 and GDP data are for 1994 (and thus precede the wage data). At any rate this still leaves, however, the local wage rate itself as endogenous variable (see \(W_i\) in equation (7)). To check for this we have experimented with instrumental variables (IV) in our estimation (not shown here, but available upon request). It is difficult to find good instruments and we (inter alia) used the size of districts, the size of the district’s population and the population density as instruments. The main conclusion is that these IV-estimations do not lead to different results. If we would have been able to use multiple years of observation for each variable in estimating equation (7), the time-series element of the data would allow us, as in Hanson’s work, to estimate in first differences. Estimation in first differences would allow us to deal with time-invariant, district-specific effects that may have a bearing on district-wages. This is

\(^{10}\) From a total of 441 districts we subtract the 119 city-districts and this gives us 322 country districts. Many of these 322 country districts are very small. In order to arrive at a geographical unit that is more in line with that of the city-district we decided not to use the 322 corresponding Kreise but to use a
not possible in our cross-section setting. With respect to the geographical unit of analysis, in our estimations the left and right hand side variables are both measured at the district level. In Hanson (1998, 1999) or Roos (2001) the latter are typically measured at a higher level of aggregation (e.g. the US state and Bundesland level) so as to make it less likely that a shock to district wages $W_r$ has an impact on $Y_s$ or $W_s$. On the other hand, a lower level of geographical aggregation of the data makes it less likely that location-specific shocks (via the error-term in equation (7)) have an impact on the independent variables.

Related with this last observation is another estimation issue, namely that the variance of the error-term varies systematically across the various districts. To address the issue of heteroskedasticity we applied the Glejser-test and used weighted least squares (WLS)). Therefore, we estimated (via non-linear least squares, NLS) equation (7) or any other of our specifications and we then we regressed the (absolute of the) resulting residuals on the right hand side variables. A significant impact of these variables on the residuals indicates heteroskedasticity and for every specification it turned out that this is indeed something that has to be taken into account. To deal with this we therefore used weighted least squares (WLS) estimations where the weights are for each specification taken from the estimation results from regressing the absolute residuals (from the “unweighted” NLS estimation) on the right hand side variables.

4. Basic estimation results for Germany

We now turn to the attempt to estimate the structural parameters using the wage equation (7) for Germany. In doing so, we will not only be able to estimate the structural parameters $\delta$, $\varepsilon$ and $T$ (and to establish the existence of a spatial wage structure) but can also verify the so-called no-black hole condition, which gives an indication for the convergence prospects in Germany. We first present the regression results of equation (7), which incorporates the assumption that the local housing stock and GDP determine the local price of housing, and then present the results of the the wage equation (7'), in which the local land price $LP$ is a proxy for the price of housing $P$ of the theoretical model.

larger geographical unit of analysis the so called Bezirke and this reduces the 322 districts to the 37 country districts. Furthermore, this simplifies the distance matrix considerably.
\[(7') \quad \log(W_r) = k' + \varepsilon^{-1} \log\left(\sum_i Y_i L_{P_i}^{\delta - \delta} Y_i^{\varepsilon} T^{1 - \varepsilon} D_{s_r} + \text{err}ight),\]

where \(LP\) is land price.

Table 2 gives the basic estimation results for the estimation of equations (7) and (7'). We also included a dummy variable for East German districts and a dummy variable for country districts.\(^{11}\) The dummy for East German districts is motivated by the fact that wages (and labor productivity) in East Germany are lower than in West Germany.\(^{12}\)

<table>
<thead>
<tr>
<th></th>
<th>Eq.(7), with housing stock, (Number of obs. 151)</th>
<th>Eq. (7'), with land prices, (Number of obs. 146)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta)</td>
<td>2.449 (2.076)</td>
<td>0.577 (29.76)</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>3.893 (8.226)</td>
<td>3.822 (13.33)</td>
</tr>
<tr>
<td>(\log(T))</td>
<td>0.009 (9.162)</td>
<td>0.0078 (7.389)</td>
</tr>
<tr>
<td>Adjusted (R^2)</td>
<td>0.95</td>
<td>0.98</td>
</tr>
<tr>
<td>Implied values:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\varepsilon/(\varepsilon - 1))</td>
<td>1.343</td>
<td>1.3</td>
</tr>
</tbody>
</table>

\(^{11}\) Inspection of the wage data revealed that there is one very large outlier, the district of Erlangen in Bavaria, which has by far the highest wage so we included a dummy for this district as well.

\(^{12}\) In our estimations we consider Germany to be a closed economy, elsewhere (see Brakman, Garretsen and Schramm, 2000) we have checked whether the inclusion Germany’s main trading partners would influence the outcomes but this was not the case. We did not control for fixed regional endowments as f.i. climate. Hanson (1999) does control for these endowments in his study for the USA but for a relatively small country like Germany these kind of differences are assumed not to be relevant. Given the fact that we use WLS means that the adjusted \(R^2\) is biased upwards. The corresponding NLS estimations (not shown) invariably resulted in an adjusted \(R^2\) in the range of 0.6-0.7.
For the estimation of equation (7), all three structural parameters are found to be significant and they also have the correct sign thereby validating the Helpman-Hanson model. The substitution elasticity $\varepsilon$ is significant and the coefficient implies a profit margin of 34.3% (given that $\varepsilon/(\varepsilon-1)$ is the mark-up), which is fairly reasonable, although higher than found for the US by Hanson (1998, 1999). Note that the value $\varepsilon(1-\delta)$ is used to determine whether a reduction of transport costs affects spatial agglomeration of economic activity: the so called no black hole condition for the Helpman (1998) model holds if $\varepsilon(1-\delta) < 1$ (see below). The coefficient for $\delta$ is, however, implausibly large in the wage equation with the housing stock because it indicates that Germans do not spend any part of their income on housing (see equation (3)). The high value is in accordance with the findings of Hanson, who also finds that $\delta$ is large for the USA (above 0.9 and in some cases also not significantly different from 1). Finally, the transport cost parameter has the expected sign and is highly significant. All in all, the estimation results provide some support for the idea of a spatial nominal wage structure, to see this substitute the estimated coefficients into wage equation (7) and one can see how ceteris paribus the presence of nearby large markets (hence low $T$ and high $Y$) increases wages in district $j$. Given the fact that we find that $d$ is clearly not significantly lower than 1, $H$, does not exert an impact on wages, but $W$, does. The first column of Table 2 also shows whether or not the no black hole condition is met. It is indeed the case that $\varepsilon(1-\delta)<1$, although not significantly (except for the case in which $\delta$

\begin{tabular}{|c|c|c|}
\hline
$\varepsilon(1-\delta)$ & -5.46 & 1.63 \\
\hline
(-1.5)$^*$ & (3.6)$^*$ & \\
\hline
\end{tabular}

Estimation method: weighted least squares (WLS); results for the constants and the east German dummy ($D_{east}$) and country dummy ($D_{country}$) are not reported; t-statistic between brackets; $^*$ $H_0$: $\varepsilon(1-\delta) = 1$.

\begin{itemize}
\item In Krugman (1991) the no black hole condition is met if $\varepsilon/(1-\delta) > 1$. Helpman (1998) shows how this difference is ultimately due to the fact that the spreading force in the Krugman model is a homogeneous tradable good (the agricultural good) whereas in the Helpman model it is a homogeneous non-tradable good (housing which is in fixed supply) is responsible for this difference.
\item Restricting $d$ to actual values of the share of income spent on non-tradable services (or non-tradable housing services) has virtually no impact on the estimated size and significance of the transport costs $T$, or on the explanatory power of the estimated equation, which is still able to explain 46% of the variance in wages, as compared to 48% in the unrestricted specification. A likelihood ratio test indicates that the restricted model has to be rejected as being inferior compared to the unrestricted model.
\end{itemize}
is fixed, see however footnote 14). This implies that agglomeration is not inevitable if transport costs can be sufficiently reduced. For Germany this seems to indicate that a lowering of transport costs might lead to more even spreading of economic activity, which is good news for the peripheral districts, the bulk of which is located in Eastern Germany. In the Helpman-Hanson model if $\epsilon(1-\delta)>1$, this means that a region’s share of manufacturing production is a function of its (fixed) relative housing stock only (Helpman, 1998, p. 40).

The second estimation shown in Table 2 uses land prices instead of the housing stock which is more in line with theoretical model since land prices are a proxy for housing prices. From Table 1 we already know that land prices (Baulandpreise), are higher in districts near to high GDP districts. An estimation of wage equation (7') with these price data provides a more direct test of the Helpman-Hanson model, because the influence of agglomeration on prices of local non-tradables is determining the strength of the spreading force in the Helpman-Hanson model. Column 2 of Table 2 gives the estimation results for 146 districts. The three structural parameters are again clearly significant. The main difference with the previous estimation is that the d-coefficient is now found to be lower. The latter is especially relevant since now we find that d is significantly smaller than 1 which indicates that a significant part of income (1-0.57) is indeed spent on housing and that the housing sector can indeed act a spreading force (the estimated share of income spent on manufactures in Germany of 0.57 is more in line with the actual share of 0.68). The other main difference is that the “no black hole condition” is not met ($\epsilon(1-d)=1.63>1$), which would imply that the spatial distribution of economic activity (and hence of district wages) only depends on the (fixed) distribution of the housing stock and that it would not depend on the level of transportation costs at all.

5. **Real Wage Differences and Wage Inflexibility**

The assumption of real wage equalization (recall equation (6)) boils down to imposing a long-run equilibrium and this (implicitly) implies a sufficient degree of labor mobility and wage flexibility. In general, the requirement that interregional real wages are equal by
assumption is not very appealing because it always assumes that the economy is in a long-run equilibrium. Furthermore, specifically in the German case this assumption seems at odds with the stylized fact that (real) wages differed between eastern and western German regions at the start of the reunification process. Another problem with the application of wage equation (7) to a European economy like Germany is that the assumption of wage flexibility is too strong.

In this section we first estimate a wage equation and the structural parameters without invoking real wage equalization. We do so by deriving a wage equation that is based on a reduced form of equations (2) and (3). For this purpose we simplify the price index defined in equation (3). We focus on two prices: the price in region $r$ of a manufactured good produced in region $r$ and the average price outside region $r$ of a manufactured good produced outside region $r$. For the determination of the simplified local price index for manufactures it also necessary to have a measure of average distance between region $r$ and the regions outside. The distance from the economic center is an appropriate measure. This center is obtained by weighing the distances with relative $Y$. The economic center of Germany turns out to be Landkreis Giessen (near Frankfurt), which is in the state of Hessen, West Germany. Equation (3) now becomes:

\[
I_r = [\lambda_r W_r^{1-\epsilon} + (1 - \lambda_r)\left(\bar{W}_rD_{r,\text{center}}^{1-\epsilon}\right)]^{1-\epsilon}Y^{\epsilon},
\]

where $\bar{W}_r$ is the average wage outside region $r$, $D_{r,\text{center}}$ is the distance from region $r$ to the economic center, and weight $\lambda_r$ is region $r$'s share of employment in manufacturing, which is proportional to the number of varieties of manufactures.

This simplified price index makes it possible to directly estimate the wage equation. Since we apply the wage equation to Germany we also take into account that the marginal productivity of labor (MPL) in East Germany is lower than in West Germany. A uniform level of MPL in the West, $\theta_{\text{west}}$, and the East $\theta_{\text{east}}$, is assumed but the MPL of the East is

\[\text{For 1 East German city district and 5 West German city districts there are no data on land prices. So they are excluded, except for Hamburg, which is also a (city) state.}\]
lower than the MPL in the West. Incorporating this difference means that the wage equation (2) and the simplified price index equation (3’) change into:

\[(2') \quad W_r = \text{constant} \cdot \left( \frac{\theta_{west}}{\theta_r} \right)^{\frac{(1-\varepsilon)^{1/\varepsilon}}{\varepsilon}} \left[ \sum_{s} Y_s \left( T^{D_{rs}} \right)^{1/\varepsilon} I_s^{1-\varepsilon} \right]^{1/\varepsilon} \]

\[(3'') \quad I_r = \left[ \lambda_r \left( W_r \frac{\theta_{west}}{\theta_r} \right)^{1-\varepsilon} + (1 - \lambda_r) \left( \frac{\theta_{west}}{\theta_r} \right)^{1-\varepsilon} \right] \]

Equation (3'') is finally substituted into (2’), which gives the reduced form of the equilibrium wage equation without having to invoke real wage equalization in order to approximate (3).

The equation to be estimated is:

\[(7'') \quad \log(W_r) = \kappa_0 + \varepsilon^{-1} \log \left[ \sum_{s} Y_s \left( T^{D_{rs}} \right)^{1/\varepsilon} I_s^{1-\varepsilon} \right] \]

where

\[I_r^{1-\varepsilon} = \left[ \lambda_r \left( W_r (1 + \kappa_i D_{east}) \right)^{1-\varepsilon} + (1 - \lambda_r) \left( \frac{\theta_{west}}{\theta_r} \right)^{1-\varepsilon} \right] \]

and where \(D_{east} = \) dummy variable which equals 1 if \(r\) is an East German district.

The first column of Table 3 shows the regression results of estimating equation (7’’). The parameter \(\kappa_1\) is set equal to zero, as it turned out to be not significantly different from zero.

\[\phi_{rs} = \phi > 1, \quad \text{if } r \text{ is West and } s \text{ is East, and } \phi_{rs} = 1/\phi < 1, \quad \text{if } r \text{ is East and } s \text{ is West. } \phi \text{ represents the real-} \]

---

16 For each region \(r\) the weighted average distance to the other regions \(\sum_{j} \text{weight}_{ij} D_{rs}\) is calculated, using \(\text{weight}_{ij} = Y_j / \sum_{j} Y_j\). The region with the smallest average distance is the economic centre.

17 Employment in a typical Western firm in a typical Western region \(r\) for the production of manufacturing variety \(i\) is \(\alpha + \beta x_{ir}\), where \(\alpha\) is the fixed cost parameter and \(\beta\) is the marginal costs parameter. Employment in a typical Eastern firm in a typical Eastern region \(r\) is \(\alpha + \beta x_{ir} (\theta_{west} / \theta_{east})\). We thus assume that marginal labor costs in East Germany are higher than in West Germany which is the same as assuming that MPL_{west} > MPL_{east}. Sales of a firm located in region \(r\) equals total demand for its product. Dropping subscript \(i\) for the individual firm:

\[\frac{(\varepsilon - 1)\alpha}{\beta (\theta_{west} / \theta_r)} = \sum_{s} Y_s \left( T^{D_{rs}} \right)^{1/\varepsilon} I_s^{1-\varepsilon} \]

Which gives (2’) above, where \(\theta_{west}/\theta_r = 1\) if \(r\) is in West, and \(\theta_{west}/\theta_r > 1\), if \(r\) is in East. Ideally, one would like to use district-data on productivity here, see for instance Funke and Rahn (2000).

18 We also captured the possibility of no real wage equalization in a more simple way by sticking to wage equation (7) and by allowing only for a real wage differential between on the one hand western and on the other hand eastern German regions by changing equation (6) into

\[\frac{W_r}{P_{r}^{1-\delta}} I_r^\delta = \frac{W_r}{P_{s}^{1-\delta}} I_s^\delta \phi_{rs},\]

where \(\phi_{rs} = \phi > 1\), if \(r\) is West and \(s\) is East, and \(\phi_{rs} = 1/\phi < 1\), if \(r\) is East and \(s\) is West. \(\phi\) represents the real-
implying that the productivity difference was not significant. An additional advantage of equation (7'') compared to the basic wage equation (7) is that the share of income spent on manufactures \( d \) (which we thus found to be rather large in our initial estimation in Table 2) does not need to be estimated now because the equilibrium condition for the housing market (equation (5)) is no longer needed to estimate the wage equation.

The results for equation (7'') in Table 3 show that the distance parameter is significantly positive, and virtually identical to previous estimates, and the same holds for \( \varepsilon \) which indicates the robustness of the estimated parameters with respect to the specifications chosen. Again, see equation (7''), the results support the notion that nominal wages in district \( r \) are higher if this region has a better access (in terms of distance) to larger markets.

<table>
<thead>
<tr>
<th>Table 3 Estimating equation (7'') and equation (8)</th>
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<tr>
<td>( \varepsilon )</td>
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<tr>
<td>LogT</td>
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<tr>
<td></td>
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<td>( C_0 )</td>
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<tr>
<td></td>
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<tr>
<td>( C_1 )</td>
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<tr>
<td></td>
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<tr>
<td>( C_2 )</td>
</tr>
<tr>
<td></td>
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<tr>
<td>Adjusted ( R^2 )</td>
</tr>
</tbody>
</table>

Estimation method for equation (7''): WLS; estimation method for equation (8): NLS; number of observations 151; results for constants and, for equation (7'), the east German dummy \( D_{east} \) and the country district dummy \( D_{country} \) are not reported.; t-statistic between brackets.

wage gap between East and West Germany (incomplete real-wage equalization). It turned out that \( \phi = 1.406 \) (t-value 4.768) thereby validating that real wages are higher in western German regions.
The estimation results for Germany provide support for the Helpman-Hanson model in the sense that the key model parameters are found to be significant. Given the coefficients and in line with equilibrium wage equation (2), our estimations of wage equations (7), (7') and (7'') illustrate that $W_r$ is higher if district $r$ is situated more closely to regions with a relatively high $Y$. We thus find confirmation for a spatial wage structure for Germany: regional wages become lower the further one moves away from manufacturing centers. To some extent this is a surprising result. Certainly compared to the case of the USA, the German labor market is considered to be rigid meaning that, for whatever institutional reason, interregional wages are set at the same level.\textsuperscript{19} For a country like Germany one might thus very well expect that the spatial distribution of $Y$ does \textit{not} get reflected in spatial wage differences (see Puga, 2001 for this assertion for Germany).\textsuperscript{20}

To investigate the relevance of the last observation we turn to regional employment. Now assuming interregional nominal wage equalization we test if we can observe a spatial employment structure under the restriction that $W=W_r=W_s$ due to centralised wage setting. Hanson (1998, 1999) estimates a spatial employment structure based on a simple market potential function. In the Appendix we derive the employment equation (8) from the theoretical model:

$$
\log \left( \frac{L_r}{\text{area}_r} \right) = \text{constant} + \log \left( \sum_{s=1}^{K} e^{-c_0 D_{rs} Y_s} \right) + c_1 D_{east} + c_2 D_{country}
$$

$L_r =$ employment in district $r$ measured in hours of employment in the manufacturing and mining sector scaled by the size of district $r$ (in km$^2$), data for 1995;

The constant, $c_0$, $c_1$ and $c_2$ are to be estimated; See the Appendix for the derivation that $c_0 = (\varepsilon - 1) \log(T)$, and $c_1 D_{east} = (\varepsilon - 1) \log(\vartheta_1)$, with $\vartheta_1$ being a measure of the productivity gap between East and West Germany.

\textsuperscript{19} One might expect that the massive transfers between western and eastern Germany are also an important institutional feature to take into account. We checked for this by using personal income which includes transfers) instead of GDP as a measure for the variable $Y$. Also, we included the ratio of each district’s GDP to personal income as an additional explanatory variable in equation (7). However, this had virtually no impact on the estimation results for the key model parameters as presented in Table 2.

\textsuperscript{20} Interregional wage differences are for instance not feasible if a union ensures centralised wage setting that is, irrespective of regional economic conditions, $W=W_r$ (see Faini, 1999). Centralised wage setting (at the industry level) is a tenet of the German labor market, see also Appendix 2.
The second column of Table 3 shows that we can confirm the existence of a spatial employment structure because of the sign and significance of the $c_0$ coefficient which implies that employment in region $j$ is higher if this regions is situated more closely to economic centers. Note that the $c_1$ and $c_2$ coefficients are also significant, indicating a lower employment in East German and country districts (given that heteroskedasticity was not found to be an issue, results in Table 3 are based on a NLS-estimation).

6. Evaluation of the estimations

So far, the estimation results provide some support for the Helpman-Hanson model. This raises the question how well the model performs against alternative models that also include the possibility of a spatial wage structure. In this section we compare our new economic geography model with two simple alternative models, a simple market potential function and a wage curve. The market potential function encompasses a wide range of theoretical approaches and as such represents a summary of alternative explanations, see Harrigan, 2001. Equation (1) is an example of a market potential function and here we use the same specification but now with wages as the dependent variable, see equation (1').

\[
(1') \quad \log (W_r) = \kappa_1 \log \left( \sum_s Y_r e^{-k_2 D_{rs}} \right) + \kappa_3 D_{east} + \kappa_4 D_{country} + \text{constant}
\]

where $W_r =$wages in district $r$, $Y_r =$personal income in district $r$, $D_{rs} =$distance between districts $r$ and $s$, $D_{east}$ and $D_{country}$ are two dummies for eastern-German and country districts respectively.

The wage curve, see equation (9), states that regional wage differences reflect regional differences in GDP or unemployment but it does not include geography in the sense that distance between regions is used as an explanatory variable (Blanchflower and Oswald, 1994).

\[
(9) \quad \log (W_r) = \beta_1 \log (y_r) + \beta_2 \log (U_r) + \text{constant}
\]

where $W_r =$wages in district $r$, $y_r =$GDP per capita in district $r$, $U_r =$unemployment in district $r$. 
We proceed as follows. We first estimate the market potential function (1') and the wage curve (9) and we then compare the models. Table 4 gives the estimation results for (1') and (9) and also gives the Akaike information criterion which is used for model selection.

<table>
<thead>
<tr>
<th>Table 4 Alternative models</th>
<th>Market potential equation (1')</th>
<th>Wage curve (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_1$</td>
<td>0.167</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(9.359)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>0.261</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.447)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_3$</td>
<td>-2.389</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-8.778)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_4$</td>
<td>-0.589</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-12.063)</td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td></td>
<td>0.509</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(11.402)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td></td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.152)</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.98</td>
<td>0.91</td>
</tr>
<tr>
<td>Akaike information criterion (AIC)</td>
<td>-0.452</td>
<td>-0.282</td>
</tr>
<tr>
<td>AIC for eq. (7), with housing stock</td>
<td></td>
<td>-0.170</td>
</tr>
<tr>
<td>AIC for eq. (7'), with land prices</td>
<td></td>
<td>-1.054</td>
</tr>
<tr>
<td>AIC for eq. (7''), with simplified price index (reduced form)</td>
<td></td>
<td>-0.067</td>
</tr>
</tbody>
</table>

# obs.=151, estimation method: weighted least squares (WLS), result for constant not reported, t-statistic between brackets. Results for Akaike info criterion are based on non linear least squares (NLS) estimations, this criterion requires that the dependent variables are the same which is not the case with WLS.
Both the market potential function and the wage curve give the expected results and thus also give rise to a spatial wage structure. The parameters are significant and have the right signs. A low value for the AIC test statistic is preferred over a higher one. It turns out that according to this criterion the Helpman-Hanson wage equation with land prices is to be preferred. With respect to the wage equation with the housing stock or our simplified price index the conclusion is, however, that the market potential function (1’) must be preferred. The wage curve is never preferred which implies that wage equations that include distance perform better. Given the fact that the wage equation with land prices is more in line with the theoretical model, one should opt for this specification. However, our estimation results also indicate that it is too soon to dismiss alternative geographical approaches summarized by the market potential function because in two out of three cases it performs better.  

Furthermore, due to data limitations we could only perform cross-section regressions for the Helpman-Hanson model. This limitation does not hold for the market potential approach. A first indication that a spatial wage structure exist over time in Germany was found by estimating equation (1’) for 1985 (only western Germany) and 1990 (the year of the re-unification) as well (not shown here). The main conclusion is that a spatial wage structure is found for all three years.

7. Conclusions

The recent advances in the field of new economic geography have increased our understanding of spreading and agglomerating forces in an economy. Empirical testing, however, is difficult. Not only because the core models are characterized by multiple equilibria, but also because the lack of specific regional data makes short-cuts inevitable. In this paper we have tried to find evidence whether or not new economic geography models

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21 Using a similar test, the Schwarz-criterion, Hanson (1999) finds for the case of the USA that wage equation (7), estimated in first differences, has always to be preferred compared to the market potential function.

22 We estimated three cross-section regressions (for 1985, 1990, 1995), a regression in first differences and a pooled regression. The cross-section estimations show that the $\kappa_1$ and $\kappa_2$ coefficients (though always highly significant) do change somewhat over time with the latter actually increasing from 1990 to 1995. The significance of a time trend in the pooled regression showed that German wages themselves are also not constant over time and are certainly determined by additional factors than merely the explanatory variables included in equation (9).
are in principle able to describe the spatial characteristics of an economy; here Germany. The answer basically is, yes. Using data for German districts we find that the so-called Helpman-Hanson model confirms the idea of a spatial wage structure. We use the Helpman-Hanson model because it incorporates the fact that agglomeration of economic activity increases the prices of a local (non-tradable) service, housing. This leads to less extreme outcomes than the core model of the new economic geography.

We modify and extend the work by Hanson (1998, 1999) in three ways. First, by using data on land prices which is more closely related to the idea that the price of a non-tradable acts as a spreading force then the housing stock. Second, we drop the assumption of real wage equalization and finally, investigate the implications of wage inflexibility by investigating the existence of a spatial employment structure. In general we not only find support for a spatial wage structure but also for relevance of the key structural parameters of the theoretical model. A comparison of our model with two alternative explanations of a spatial wage structure indicates that our model specification with land prices is to be preferred. However, this comparison also shows that it is too early to conclude that the new economic geography approach systematically outperforms alternative explanations of the spatial distribution of wages. A straightforward next step is to analyze the development of a spatial wage structure over time using a new economic geography approach.
Appendix: Derivation of the Employment Equation (8)

With centralised wage-setting:
\[ W_r = W_s = W \]

Assume a productivity gap between East and West:
\[ L_{ir} = \alpha + \theta \frac{\partial}{\partial r} \theta_{west} x_{ir} \]

where \( L_{ir} \) is employment in firm \( i \) in region \( r \), \( x \) is output, \( \theta_{west} \) is the marginal productivity of labour in West Germany, and \( \theta \) is 1 if region \( r \) is in West and \( \theta \) is \( \theta_{west}/\theta_{east} > 1 \), if region \( r \) is in East.

Free entry and exit and using the no-profit condition leads to the equilibrium output for firm \( i \) in region \( r \) (see Brakman, Garretsen and van Marrewijk, 2001, pp 78-79):
\[ x_{ir} = \frac{\alpha (e - 1)}{\theta \frac{\partial}{\partial r} \theta_{west}} \]

Substituting the expression for equilibrium output into the above employment equation gives labour demand at the firm level:
\[ L_{ir} = \alpha e \]

We can use this to express output in units of labour:
\[ x_{ir} = \left( \frac{e}{e - 1} \right) L_{ir} \frac{\partial}{\partial r} \theta_{west} \]

Summing over all firms \( i \) and rewriting this last equation employment in each region \( L_r \) can be expressed in logarithms as follows:
\[ \log(L_r) = \text{constant} + \log(\partial_r) + \log(x_r) \]

The \( x_r \) in this equation is the equilibrium output for region \( r \). The equilibrium demand facing each firm \( i \) is
\[ x_d = \frac{p^{\varepsilon}}{I^{1-\varepsilon}} \delta Y \]

Summing over all firms, using the optimal pricing rule is \( p = \frac{\varepsilon}{\varepsilon - 1} \theta \theta W \) and taking transport costs into account gives:
\[ x_r = \sum_{i=1}^{n} \left[ \frac{\varepsilon}{\varepsilon - 1} \frac{W_r T^{D_{rs}} (\theta \theta_{west} \theta_{r})}{I_s} \right] T^{D_{rs}} \frac{Y_s}{I_s} \]

where \( T \) is transport costs, and \( D_{rs} \) is the distance between regions \( r \) and \( s \), \( I \) is the price index of manufactures.

Substituting this expression into:
\[ \log(L_r) = \text{constant} + \log(\partial_r) + \log(x_r) \]

gives
\[
\log(L_r) = \text{constant} + \log(\hat{\theta}_r) + \sum_{s=1}^{k} \left[ \left( \frac{\varepsilon}{\varepsilon - 1} \left( W_s T^D_{r_s} \left( \theta_{r_west} \hat{\theta}_s \right) \right) \right)^{-\varepsilon} T^D_{r_s} \delta \frac{Y_s}{I_s} \right] = 0
\]

\[
\log(L_r) = \text{constant} + \log(\hat{\theta}_r) - \varepsilon \log(W_r) + \log \left( \sum_{s=1}^{k} \left[ \left( \hat{\theta}_s \right)^{-\varepsilon} \left( T^D_{r_s} \right)^{1-\varepsilon} Y_s I_s^{\varepsilon-1} \right] \right)
\]

because of the assumption of uniform nominal wages: \( W_r = W \), and

\[
\log(L_r) = \text{constant} + \log(\hat{\theta}_r) + \log \left( \sum_{s=1}^{k} \left[ \left( \hat{\theta}_s \right)^{-\varepsilon} \left( T^D_{r_s} \right)^{1-\varepsilon} Y_s I_s^{\varepsilon-1} \right] \right)
\]

For an East German district \( r \) the employment equation is:

\[
\log(L_r) = \text{constant} - (\varepsilon - 1) \log(\hat{\theta}_r) + \log \left( \sum_{s=1}^{k} \left[ T^D_{r_s} \right]^{1-\varepsilon} Y_s I_s^{\varepsilon-1} \right)
\]

For a West German district \( r \) the employment equation is:

\[
\log(L_r) = \text{constant} + \log \left( \sum_{s=1}^{k} \left[ T^D_{r_s} \right]^{1-\varepsilon} Y_s I_s^{\varepsilon-1} \right)
\]

To arrive at the specification to be estimated add a dummy variable that is 1 for East German districts and 0 for West German districts, the sign should be negative. Following Hanson we scale district employment by the variable \( \text{area}_r (= \text{km}^2 \text{ of a district}) \) in order to account for the differences in district size in the sample. So the dependent variable becomes \( L_r/\text{area}_r \). Using the long-run equilibrium in which real wages are equalized means that price indices of manufactures are equalized. The employment equation now becomes:

\[
\log(L_r) = \text{constant} + \log \left( \sum_{s=1}^{k} \left[ T^D_{r_s} \right]^{1-\varepsilon} Y_s I_s^{\varepsilon-1} \right) + c_1 D_{east}
\]

As this equation shows, it is not possible to estimate the structural parameters \( \varepsilon \) and \( T \) separately. So the equation in the main text that has actually been estimated is:

\[
\log \left( \frac{L_r}{\text{area}_r} \right) = \text{constant} + \log \left( \sum_{s=1}^{k} \left[ e^{-c_0 D_{r_s}} Y_s \right] \right) + c_1 D_{east}
\]

where the constant, \( c_0 \) and \( c_1 \) are to be estimated.

Note that \( c_0 = (\varepsilon - 1) \log(T) \), and \( c_1 D_{east} = (\varepsilon - 1) \log(\hat{\theta}_r) \).
References
Brakman, S., H. Garretsen, and Ch. van Marrewijk (2001), *New Economic Geography in Germany: Testing the Helpman-Hanson Model*, *HWWA Discussion Paper*, Hamburg (see [http://www.hwwa.de/hwwa.html](http://www.hwwa.de/hwwa.html)).
Hanson, G.H. (1999), Market Potential, Increasing Returns, and Geographic Concentration, *mimeo*, University of Michigan (revised version of Hanson, 1998).


