Answers to questions for *An Introduction to Geographical Economics*

Chapter 3 The core model of geographical economics

**Question 3.1**
From introductory micro-economics we know that the condition for profit maximization for a firm is $MC = MR$, that is marginal costs equals marginal revenue. Under perfect competition this condition implies that $MC = p$, that is marginal cost is equal to the price of a good (marginal cost pricing). Now use Figure 3.7 to show that with the average cost curve in the core model (use (3.10)), marginal cost pricing always results in a loss for the firm; implying that imperfect competition is the dominant market form.

**Answer 3.1**

\[ \text{p} \quad \text{mr} \quad D \]

Above the picture is drawn again: p=price, mc=marginal costs, ac=average costs, D=demand curve, mr=marginal revenue associated with D. The average cost curve is a declining function of x, due to the fixed costs. Because the marginal costs are constant the ac-curve lies everywhere above the mc-curve. A thought experiment is illuminating.
What happens if a firm behaves like a profit maximizing firm in a perfect competitive market and sets its price equal to marginal costs, \( p = mc (= mr \text{ in the perfect competition case}) \)? The demand curve gives the sales associated with \( p = mc \). The average cost curve gives the average cost corresponding with this level of production. Now, because the average cost curve is always above this price level (which equals \( mc \)) the grey area gives the total loss of our firm with this pricing strategy. So, perfect competition is not consistent with this form of increasing returns to scale. What market form is? One option could be a monopoly. Profit maximization in this case (now the firm can influence the price level) equates \( mr = mc (\neq p) \). From the demand curve we can infer the price that corresponds with this level of sales. The dotted area gives total profits for the monopolist. So, a monopoly could be consistent with this form of increasing returns to scale. Other forms of imperfect competition can also be consistent, and as we have seen monopolistic competition is a very rewarding market form for the Economic Geography approach.

*Question 3.2.*

Start again from the example in section 3.2, but now assume that each firm has the possibility to open a second plant in the other region. Each firm minimizes the combined costs of setting up a second plant and transportation costs. Suppose setting up a firm costs 2 units. Decide where to locate given the location of the other firms.

a. If all other firms have a single firm in South, what is optimal for our firm?

b. Suppose all firms have two plants, one in each location, what is optimal for our firm?

*Answer 3.2*

Location and Transportation costs

<table>
<thead>
<tr>
<th></th>
<th>Firm in N</th>
<th>Firm in S</th>
</tr>
</thead>
<tbody>
<tr>
<td>All firms in N</td>
<td>0 + 2 = 2 (to farmers in S)</td>
<td>4 + 4 = 8 (to workers and farmers in N)</td>
</tr>
<tr>
<td>All firms in S</td>
<td>4 + 2 = 6 (to workers and farmers in S)</td>
<td>0 + 4 = 4 (to farmers in N)</td>
</tr>
<tr>
<td>25% firms in N 75% firms in S</td>
<td>3 + 2 = 5 (to workers and farmers in S)</td>
<td>1 + 4 = 5 (to workers and farmers in N)</td>
</tr>
</tbody>
</table>
The table above repeats table 3.2 from the example. From the example we already learned that if all firms are located in the South (S) it is optimal for a firm also to locate in the South. But, can it be worthwhile to start business in the North (N) if set-up cost of a subsidiary is 2? In this case it is because the firm can forego transport costs of 4 by investing 2 to set up shop in N. If all firms were located in N a firm would be indifferent (why?).

b. With an equal division of workers over both regions, the largest market is in N, because most of the farmers are located there. That by itself makes it an attractive location. Transport cost to ship to S in this case are 2 (farmers in S)+2(workers in S)=4 >2 (set up cost of a second plant in S). So our firm also decides to have two plants, one in each location.

**Question 3.3**

Suppose we start with a situation of complete agglomeration of manufacturing production in region 1, that is $\lambda_1 = 1$. Without calculating the equilibrium values explicitly one might suspect that $W_1 = 1$ is the equilibrium value in this case. Substituting this in the income and price equations we find:

$$Y_1 = \frac{(1 + \delta)}{2}$$
$$Y_2 = \frac{(1 - \delta)}{2}$$
$$I_1 = 1$$
$$I_2 = T$$

Using these values, $W_1 = 1$ is indeed an equilibrium value for wages in region 1, as can be verified from (3.30). Real wages also equal 1 in region 1. Calculate under which condition this is always a long run equilibrium no matter how large transportation costs become. Hint: use the expression for real wages in region 2 for this case and let $T$ become arbitrarily large. Show that this only happens if $(\varepsilon - 1 - \varepsilon \delta) < 0$.

**Answer 3.3**

We only have to find out whether real wages are smaller than one or larger. Inserting the values for income and price indices into the real wage equation (3.8) gives:
This equation gives the real wage for region 2. The first term on the right hand side states that real wages in a region tend to be lower the greater the distance to the import markets. In this case region 2 has to import all its manufactured goods from region 1, the manufacturing center in this set-up. Because $T > 1$, this term is smaller than one, which makes region 2 a less attractive place for manufacturing workers. The term between brackets is the (nominal) wage rate, which is consistent with zero profits for each firm. Both income terms are weighted with transportation costs. The income in region 1 is weighted by $T^{1-\varepsilon} < 1$. For a firm in region 2 this means that income in the other region is of less importance due to the transportation disadvantage of a firm in region 2 relative to a firm in region 1. The income in region 2 is weighted by $T^{\varepsilon-1} > 1$. This is the opposite effect for a firm in region 2; income of this region is of more importance to him than to a firm based in region 1, due to the location advantage of a region 2 firm compared to a firm located in region. For $T = 1$, i.e. if there are no transportation costs, also $\bar{w}_2 = 1$. In the “free” trade equilibrium location does not matter. What happens to $\bar{w}_2$ if transportation costs rise. Differentiating the real wage equation above and evaluating it around $w_2 = 1$, and $T = 1$, we find:

$$\frac{dw_2}{dT} = \frac{\delta (1 - 2\varepsilon)}{\varepsilon} < 0,$$

for small transportation costs. For high transportation costs the analysis is slightly more complicated. Inspecting (*) we immediately see that the first term between brackets becomes arbitrarily small, if $T$ becomes arbitrarily large. But what happens to the second term, depends on the sign of $\varepsilon - 1 - \varepsilon \delta$. If this term is smaller than zero, this term also becomes arbitrarily small and complete agglomeration is always sustainable. However, if $\varepsilon - 1 - \varepsilon \delta > 0$, the second term in equation (*) becomes arbitrarily large if $T$ becomes arbitrarily large. So, complete agglomeration is not sustainable for large values of $T$.\textsuperscript{1} But, we also established that for relative small values

\textsuperscript{1} We abstract from the complication that in fact complete agglomeration is always an equilibrium. The discussion in the text assumes that it it possible to compare the manufacturing wages in the two regions
of \( T \), complete agglomeration is always sustainable. In chapter 4 we will return to the precise meaning of \((\varepsilon - 1) - \varepsilon \delta > 0\).

**Question 3.4**

Suppose a monopolistic producer located in region 1 can either sell in region 1 or in region 2. Let \( p_{11} \) (\( p_{12} \)) be the price charged in region 1 (respectively in region 2), and let \( x_{11} \) (\( x_{12} \)) be the demand in region 1 (respectively in region 2). Obviously, the demand functions depend on the price charged in either region. Production requires only labor as an input, which is paid wage rate \( W_1 \), and benefits from internal returns to scale, using \( \alpha \) fixed labor and \( \beta \) variable labor. Finally, there are (iceberg) transport costs: the firm must produce \( T x_{12} \) units to ensure \( x_{12} \) can be sold in region 2, with \( T > 1 \). The firm’s profit \( \pi \) and demand functions \( x_{11} \) and \( x_{12} \) (\( \varepsilon > 1 \), while \( Y_1 \) and \( Y_2 \) are constants) are given below:

\[
\pi = p_{11} x_{11} + p_{12} x_{12} - W_1 (\alpha + \beta x_{11} + \beta T x_{12})
\]

\[
x_{11} = p_{11}^{1+\varepsilon} Y_1; \quad x_{12} = p_{12}^{1+\varepsilon} Y_2
\]

First, give some comments on the profit function above. Second, substitute the demand functions in the profit function. Third, determine what the optimal prices \( p_{11} \) and \( p_{12} \) are, that is solve the profit maximization problem. Fourth, show that \( p_{12} = T p_{11} \), that is the optimal price charged in region 2 is exactly \( T \) times higher than the optimal price charged in region 1.

**Answer 3.4**

First, we note from the specification of the profit function that the entrepreneur can independently choose the price charged in each region (see also the remark at the end of this answer). Second, if we substitute the demand functions in the profit function we get:

\[
\pi = p_{11}^{1+\varepsilon} Y_1 + p_{12}^{1+\varepsilon} Y_2 - W_1 (\alpha + \beta p_{11}^{1+\varepsilon} Y_1 + \beta T p_{12}^{1+\varepsilon} Y_2)
\]

Third, we can differentiate this profit function with respect to the price to be charged in either region and equate the result to zero to get the first order conditions for profit maximization (that is to determine the optimal price to be charged in both regions):

\[\ldots\]
\[
\frac{\partial \pi}{\partial p_{11}} = 0 \quad \Rightarrow \quad (1-\varepsilon)p_{11}^{\varepsilon+1}Y_1 + \beta W_1 p_{11}^{\varepsilon+1}Y_1 = 0
\]
\[
\frac{\partial \pi}{\partial p_{12}} = 0 \quad \Rightarrow \quad (1-\varepsilon)p_{12}^{\varepsilon+1}Y_2 + \beta TW_1 p_{12}^{\varepsilon+1}Y_2 = 0
\]
Solving these first order conditions determines the optimal prices to be charged:
\[
p_{11} = \frac{\varepsilon}{\varepsilon - 1} \beta W_1; \quad p_{12} = \frac{\varepsilon}{\varepsilon - 1} \beta TW_1
\]
Fourth, it immediately follows that the optimal price charged in region 2 is exactly T times higher than the price charged in region 1. This is important because it implies that consumers or other economic agents are not involved in parallel imports into region 2, as there are no profits in this activity. The fact that the price elasticity of demand is the same in the two regions is crucial in this respect. If the price elasticity of demand is not the same in the two regions, we must either assume that markets are segmented, such that the firm can determine the optimal price in each region without fear of parallel imports, or that the price charged in the two regions is determined taking into account the possibility of parallel imports.

Question 3.5*
In the example of section 3.2 we showed that some equilibria are better from a welfare perspective than other equilibria. Can you show this using (3.22)-(3.24) and (3.22')-(3.24')? Assume that the farmers are not symmetrically distributed over both regions. Suppose region 1 has 1/3 of all farmers and region 2, 2/3 of all farmers. Can you show that from a welfare point of view, agglomeration in region 2 is better than agglomeration in region 1, as might be expected because region 2 potentially has the largest market.

Hint: make sure that complete agglomeration in region 1 and complete agglomeration in region 2 are both equilibria. Use the resulting equations to show that (U indicates utility)

For \( \lambda_I = 1 \) we have:
\[
U_{\lambda=1} = \delta + \frac{(1-\delta)}{3} + \frac{2}{3} (1-\delta) r^{-\delta}
\]
And for \( \lambda_I = 0 \):
\[
U_{\lambda=0} = \delta + \frac{2}{3} (1-\delta) + \frac{1-\delta}{3} T^{-\delta}
\]
A more in depth analyses of welfare is given in Chapter 4.
Answer 3.5*

What can we say about welfare? In the example we see that multiple equilibria are possible, but also that a specific equilibrium might be better than another, from a welfare point of view. From the indirect utility function in section 3.3.1 we see that welfare depends on (nominal) income and the consumption-based price index. The only income in this model is labour income. Furthermore, we have four different groups in our economy: manufacturing workers in both regions and Food producers in both regions. To give an example, we can derive welfare for each of the equilibrium situations we have studied above: complete agglomeration and the symmetrical equilibrium. For each equilibrium the economy-wide welfare is simply the sum of labour income for each group times the relevant price index for that particular group.

\[ U_{total} = \lambda \delta w_1 + \left(1 - \lambda\right) \delta w_2 + \frac{(1 - \delta)}{3} I_1^{-\delta} + \frac{2(1 - \delta)}{3} I_2^{-\delta} \]

Note, that the real income of the Food producers might also differ between the two regions, because it also matters for them whether or not they have to import manufactures. Again we can follow the procedure in the text and guess that for complete agglomeration in region 1, \( W_1 = 1 \) is indeed an equilibrium (verify this using 3.24). For complete agglomeration in region 1 we then have \( Y_1 = \frac{(2\delta + 1)}{3}, \ Y_2 = \frac{2(1 - \delta)}{3} \), and using \( I_1 = 1, I_2 = T \) and \( W_f = 1 \):

we have: \[ U_{\lambda, i} = \delta + \frac{(1 - \delta)}{3} + \frac{2}{3}(1 - \delta)T^{-\delta} \]

Similarly, for complete agglomeration in region 2 we get:

\[ U_{\lambda, o} = \delta + \frac{2}{3}(1 - \delta) + \frac{(1 - \delta)}{3}T^{-\delta} \]

Subtracting the two expressions shows that agglomeration in region 2 is better from a welfare point of view.