Survival of the fittest in cities: Agglomeration, selection, and polarisation*

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Abstract

Empirical studies consistently report that labour productivity and TFP rise with city size. The reason is that cities attract the most productive agents, select the best of them, and make the selected ones even more productive via various agglomeration economies. This paper provides a microeconomically founded model of *vertical city differentiation* in which the latter two mechanisms ('agglomeration' and 'selection') operate simultaneously. Our model is both rich and tractable enough to allow for a detailed investigation of when cities emerge, what determines their size, and how they interact through the channels of trade. We then uncover stylised facts and suggestive econometric evidence that are consistent with the most distinctive equilibrium features of our model. We show, in particular, that larger cities are both more productive and more unequal ('polarised'), that inter-city trade is associated with higher income inequalities, and that the proximity of large urban centres inhibits the development of nearby cities.

Keywords: heterogeneity; firm selection; agglomeration; income inequalities; urbanization; urban systems

JEL Classification: F12; R12

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1 Introduction

For the first time in history, human beings who live in cities account for more than half of the world population. Such concentration of population into a limited number of places is clearly reflected by the spatial distribution of economic activity: cities, or places of high human density, are locations of striking wealth creation that produce a disproportionate share of output and spawn most innovations.[1] Similar patterns are also observable at a more disaggregated level. Many empirical studies have, indeed, substantiated that average productivity is increasing in local market size and density: in any cross-section of cities, the elasticity of labour and firm productivity with respect to size or density is positive and typically in the 3% – 8% range (Rosenthal and Strange, 2004). Cities are also places *par excellence* where people acquire useful skills and where new ideas are bread through interactions and knowledge transfers.

Despite their paramount contribution to wealth creation, slums in developing countries and urban ghettos in rich ones remind us that cities are polarised: they are places where utter poverty runs alongside phenomenal wealth. The reason is that although cities attract many skilled and unskilled people and many firms, a substantial share of them fails to realise their aspirations. This is exemplified by the substantial churning that occurs in urban areas, i.e., at any given moment many firms are being created and many cease to exist.[2]

In a nutshell, cities are places that make workers and firms more productive (*'agglomeration'*), yet where failure is more likely than elsewhere (*'selection'*), thereby generating large inequalities (*'polarisation'*). While each one of these three features has been addressed individually in the literature there is, to the best of our knowledge, as yet no theory that addresses them simultaneously.[3] The first contribution of our paper is, therefore, to provide a theoretical framework that aims at filling this gap. To do so, we devise a model that blends heterogeneous managerial talent

[1] In 1990, the three Japanese main metropolitan areas (Tokyo, Osaka, Nagoya) made up for a third of the Japanese population (about 2.6% of East Asia's) but for 40% of Japanese GDP (about 29% of East Asia's; see Fujita and Thisse, 2002). Feldman and Audretsch (1999) report that in 1982, 96% of U.S. product innovation took place in large metro areas, home to about 30% of the U.S. population.

[2] On the relationship between size or density and productivity, see Sveikauskas (1975), Ciccone and Hall (1996), Syverson (2004), Combes *et al.* (in progress) and, for a survey, Rosenthal and Strange (2004). On the relationship between city size and hours worked that result from both selection and agglomeration, see Rosenthal and Strange (2008); on the division of labour and market size, see Baumgardner (1988). Glaeser *et al.* (1992), Henderson *et al.* (1995) and Audretsch and Feldman (2004) provide evidence on innovation and growth in cities; on skills in cities, see Glaeser and Maré (2001) and Bacalod *et al.* (2008). Harris and Todaro (1970), Cutler *et al.* (2005) and Glaeser *et al.* (2008) address the issue of urban polarisation; on churning, see Duranton (2007).

[3] A recent and unique exception is provided by Combes *et al.* (in progress), who embed a reduced-form agglomeration force in the Melitz and Ottaviano (2008) framework. Their focus is mostly empirical and aims at disentangling selection effects from agglomeration economies. A complementary approach is Okubo (in progress), who encapsulates the Melitz (2003) trade model into a 'new economic geography' framework *à la* Fujita *et al.* (1999b). However, by nature of its focus, this model disregards urban structure.

in the spirit of Lucas (1978), with functional forms taken from both the heterogeneous firm model put forth by Melitz and Ottaviano (2008) and the 'new economic geography' (NEG) setting from Ottaviano *et al.* (2002). The resulting framework allows us to parsimoniously analyse the various interactions between agglomeration, selection and polarisation in an urban environment with heterogeneous agents. We tackle, in particular, the questions of *when cities emerge*, how they *grow*, whether or not they are *resilient* to adverse shocks, and how they *interact* with one another through the channels of trade. Our second contribution is to show how selection and polarisation interact, and to establish that urban polarisation is a by-product of the survival of the fittest in a tough, competitive environment. Cities make the best of firms' workers because they fail the least productive of them, and the more so the bigger they are. Consequently, larger cities have higher productivity but are more unequal (see Long *et al.*, 1977, for U.S. evidence). Our last contribution is to uncover a set of stylised facts in the data that are consistent with the most distinctive equilibrium features of our model. We show, both theoretically and empirically, how cities interact with each other and influence their respective productivities and sizes. Such an exercise is, in our opinion, worthwhile since there is to date a dearth of simple testable predictions derived from the (general) equilibrium conditions of NEG models (Head and Mayer, 2004a).

Previewing our main theoretical results, we first characterise the conditions under which one city would emerge independently of others. We show, rather intuitively, that cities are more likely to emerge and to grow in size with the quality of the commuting technology (e.g., following the invention of the streetcar in the second half of the nineteenth century), the entrepreneurs' expected productivity (linked, e.g., to the quality of the institutional, legal, and eductional systems), and the magnitude of the agricultural surplus (as captured by stronger preferences for manufactures). Strikingly, income inequalities are also positively associated with these factors.[4] We further show that several equilibria may co-exist, including one without cities: whenever this occurs, the equilibrium with the largest city also has, on average, more productive entrepreneurs. In other words, large urban centres provide a tougher environment in which only the fittest survive, and *there is thus a positive equilibrium relationship between agglomeration and selection*. As a corollary, our model predicts that a relatively large proportion of urban dwellers fails to be successful in bigger cities, thus making the latter more unequal and polarised.[5] Using U.S.

[4]The mechanism we unveil in our model complements the one in Abdel-Rahman and Wang (1997), who also develop an urban model that generates income inequalities. In their model, skilled workers are horizontally differentiated only, and firms require different skills along a Salop circle; there is a finite number of firms because of production non-convexities. Workers whose skills match the needs of the firm that employs them best will earn a higher equilibrium wage (wages are set using a Nash-bargaining concept). Agglomeration economies arise because the average mis-match decreases as the density of firms along the circle rises. To the best of our knowledge, theirs is the only model that studies income inequalities in an urban setting.

[5]It is well known that the industrial mix of large cities is generally more diverse than that of smaller ones (Henderson, 1997). Duranton and Puga (2005) show that the same holds increasingly true for the 'functional mix' of cities. In both cases, these facts regard the *horizontal* diversification of cities. Here, we show that the

Core Based Statistical Area (CBSA) data, we provide some suggestive econometric evidence that strongly supports this effect.

We then extend our model to a system of cities that are linked by a transportation network. Specifically, for simplicity we first consider a symmetric equilibrium in which all cities are identical (same size and same composition of successful entrepreneurs). In that case, the comparative static results derived in the one-city setup carry over to this new environment. Having multiple cities allows us, however, to analyse how they interact through trading links. We show that lower inter-city trade costs are conducive to city formation and city growth: access to larger markets, brought about by innovations in the transportation sector (e.g., the steam engine, trucks, cargo ships, and containerisation) unambiguously increase the prospect of urbanisation and city sizes. Historically, this corresponds to the emergence and domination of trading cities like London, Genoa, Buenos Aires or, more recently, Hong Kong and Yokohama. We then study the properties of urban systems made of cities of different sizes. The positive relationship between city size and productivity readily extends to this context: larger cities provide a larger variety of goods and services to consumers, thus establishing a hierarchy that shares some features with Lösch's (1940). However, we also show that trading links may inhibit the growth of some cities through 'cannibalisation effects'. More precisely, the rise of a large, productive city might hinder the development of nearby urban centres. The reason is that, from the persepective of a given city, other cities provide both an outlet for goods and services they produce, yet they also increase competition through exports by their entrepreneurs; the net effect is negative when proximate cities are large enough to cast an 'agglomeration shadow'. Building upon the equilibrium conditions of our model, we derive a spatial econometric specification and provide some evidence consistent with this cannibalisation effect (see Dobkins and Ioannides, 2000). The presence of this effect has an important implication for the size-and-productivity literature: running a simple OLS to estimate the coefficient of city size on city productivity produces a downward bias when the spatial structure of the city system is disregarded.

Finally, we show that our framework can readily be extended to address a variety of issues. For instance, our model suggests that cities evolve from 'producer cities' in the early stages of the industrial-cum-transportation revolution into 'consumer cities' as they become part of an integrated network of trading cities (Weber, 1958; Glaeser *et al.* 2001; Tabuchi and Yoshida, 2000). We also show that income inequalities are increasing in trade openness (at least at the symmetric equilibrium), but for reasons that are quite different from those unveiled in the international trade model of Helpman *et al.* (2008). Interestingly, the data reveals that income inequalities are decreasing in market potential, and we explain how to reconcile this apparent contradiction between our theoretical predictions and the empirical findings.

The remainder of the paper is organised as follows. Section 2 presents the basic model

composition of large cities is *vertically* differentiated as well.

and derives the equilibrium conditions. Section 3 then deals with the single-city case, whereas Section 4 extends the model to multiple cities and trading networks. Section 5 deals with various asymmetric configurations, while Section 6 concludes. We relegate the most technical proofs, the guide to various calculations, as well as some extra material, to an extensive set of appendices.

2 The model

We start by sketching the model. There are Λ regions, labeled $l = 1, 2, \ldots, \Lambda$. Variables associated with each region will be subscripted accordingly. Region l has a large and fixed population L_l of ex ante undifferentiated workers. Each worker is endowed with one unit of labor that she can use either for producing a numéraire good as an unskilled worker, or for becoming a skilled entrepreneur. Becoming an entrepreneur involves a net entry or education cost and requires that the worker moves to city l which provides the adequate environment for starting a business. 'Learning in cities' is in accord with empirical evidence (Glaeser and Maré, 2001) and the fact that most universities are located there. As will become clear later, becoming an entrepreneur entails a risk of failure, in which case the agent is stuck in the city, does not produce, and consumes solely from his initial endowment. Hence, there are three types of agents: successful skilled agents in the cities (entrepreneurs); unsuccessful skilled agents in the cities; and unskilled agents who decide to stay in the rural area. A maintained simplifying assumption is that workers who choose to move to the city do not relocate once their decision to enter the urban market and to set up business has been taken, irrespective of whether they succeed or fail as entrepreneurs.[6]

There are two sectors in the economy. The first one produces a continuum of varieties of a horizontally differentiated good or service, whereas the second one produces a homogenous good. Production of the homogenous good requires no entrepreneurial skills, occurs under constant returns to scale, and takes place outside of the city (think, e.g., of agriculture). Furthermore, the homogenous good is traded in a competitive market which is perfectly integrated. Hence, its price is equalized across regions which makes this good a natural choice for the numéraire. Perfect competition ensures that marginal cost pricing prevails, which implies a unit wage everywhere as long as the homogenous good is produced in all regions, which we henceforth assume to be the case. The differentiated good is produced by the entrepreneurs using entrepreneurial skills and

[6]It is worth noting that workers have no ex ante information on their ex post productivity, i.e., there is no sorting according to skills in our model. Yet, selection and agglomeration generate higher productivity in larger cities regardless of sorting, so adding sorting to our model would reinforce our results. See Mion and Naticchioni (forthcoming), Combes *et al.* (2008) and Combes *et al.* (in progress) for evidence on sorting. It is worth pointing out that recent empirical evidence suggests that "large cities are more skilled than are small cities, but to a modest degree. The differences are smaller than are the differences in worker education across cities, which Berry and Glaeser (2005) argue are themselves not very large" (Bacolod *et al.*, 2008, p.3). We disregard issues related to sorting in this paper. See Mori and Turrini (2005) and Nocke (2006) for models of vertical city differentiation with sorting.

the numéraire good. The latter is obtained either from the entrepreneur's endowment or from the country-side.

Previewing our subsequent results, only those entrepreneurs who are productive enough survive and produce, whereas low-ability entrepreneurs leave the market immediately without setting up production at all. In other words, not everybody is equally successful. The minimum ability that entrepreneurs have to achieve to survive is an equilibrium feature of the model that we refer to as *selection*. Entry into the city occurs in reponse to economic opportunities, which depend largely on the ability threshold required for producing successfully. We view the determination of city size $H_l \leq L_l$ at equilibrium as the result of a tension between agglomeration and dispersion forces, a phenomenon that we refer to as *(net) agglomeration* for short.

2.1 Timing

There are two stages. In the first one, workers decide whether to become entrepreneurs, in which case they incur the entry cost $f^E \geq 0$ (paid in terms of the numéraire and including the opportunity cost of foregoing the unskilled wage), or to stay as uneducated workers in the country-side. Henceforth, superscript 'E' is a mnemonic for 'entry' or 'education', whereas superscript 'U' is a mnemonic for 'unskilled' or 'uneducated'. Interpreting f^E as a cost to acquiring education, this means that agents decide first whether to acquire skills or not and, if so, become urban dwellers, and only then learn their ability.[7] Living in a city gives rise to extra costs and benefits, which will be made precise below. Once the education-cum-location decision is taken, *nature* attributes to each entrepreneur a horizontal characteristic ν and a vertical characteristic c: we think about the former as the product variety (or a *type* of skill) and about the latter as her entrepreneurial ability (or her skill *level*), which is linked to the subsequent productivity of the firm. Specifically, entrepreneurs discover a variety or blueprint and nature draws the marginal cost c at which they can produce this variety from some common and known distribution $g_l(\cdot)$. Upon observing their draw c, entrepreneurs chose whether to produce or not and to which markets to sell. Entrepreneurial skills are an indivisible input and we assume that production has to be spatially integrated, i.e., the entrepreneur must colocate with the production facility. Entrepreneurs that entered market l in the first stage live and consume in city l. In the second stage, entrepreneurs set profit maximising prices and all markets clear. We solve by backward induction for subgame perfect equilibria.

[7]Note that f^E might conceivably be decreasing in city size as a result of agglomeration economies, especially if we think about it as being the costs of becoming educated. To keep the analysis manageable, we treat f^E as a parameter throughout the analysis and sometimes impose the knife-edge assumption $f^E = 0$.

2.2 Preferences, demand, and urban structure

Following Ottaviano *et al.* (2002) and Melitz and Ottaviano (2008), all agents have identical quasi-linear preferences over the homogenous good and the varieties of the horizontally differentiated good. Furthermore, each agent is endowed with $\overline{d}^0$ units of the numéraire. Varieties of the differentiated good available in region l are indexed by $\nu \in \mathcal{V}_l$. In what follows, we denote by $\mathcal{V}_{hl}$ the set of varieties produced in h and consumed in l, so that $\mathcal{V}_l \equiv \cup_h \mathcal{V}_{hl}$; by $\mathcal{V}_l^+ \subseteq \mathcal{V}_l$ the subset of varieties *effectively consumed* at equilibrium in region l; and by N_l the measure of $\mathcal{V}_l^+$ (the mass of varieties consumed in l). The subutility over the differentiated varieties is assumed to be quadratic, so that utility for a resident in region l is given by:

$$U_l^i = \kappa^i \left\{ \alpha \int_{\mathcal{V}_l} d_l(\nu)\mathrm{d}\nu - \frac{\gamma}{2} \int_{\mathcal{V}_l} [d_l(\nu)]^2 \,\mathrm{d}\nu - \frac{\eta}{2} \left[\int_{\mathcal{V}_l} d_l(\nu)\mathrm{d}\nu \right]^2 \right\} + d_l^0, \tag{1}$$

where $\alpha > 0$, $\eta > 0$, and $\gamma > 0$ are preference parameters; where d_l^0 and $d_l(\nu)$ stand for the consumption of the numéraire and of variety ν, respectively; and where $\kappa^i = 1$ if $i = E$ (the agent lives in the city) or $\kappa^i = 0$ if $i = U$ (the agent lives in the country-side). Our assumption on κ^i implies that the differentiated good is sold and consumed exclusively in the cities, whereas the homogenous good is available and consumed everywhere.[8]

Since marginal utility at zero consumption is bounded for each variety, urban dwellers will in general not have positive demand for all of them. In what follows, we assume that all urban agents have positive demand for the numéraire, i.e., $d_l^0 > 0$, which rules out income effects. A sufficient condition for this to hold is that the initial numéraire endowment $\overline{d}^0$ is large enough, which we henceforth assume to hold true.

All entrepreneurs reside in a monocentric city and, therefore, have to pay commuting costs and land rents. Furthermore, there is at most one city per region.[9] The aggregate land rent is redistributed among the urban dwellers, each of whom has a claim to an equal share of it. The urban costs in region l, when its size is H_l, are captured by the reduced-form θH_l, which

[8] This assumption is a short-cut, the purpose of which is to make the market size for the differentiated good endogenous while retaining a parsimonious model. A more elegant micro-foundation would be to assume that shipping varieties from the city to the rural areas entails some cost. Our setting may be viewed as the case where this cost is prohibitive. Though this is a strong assumption, it is worth keeping in mind that trade between major cities (located often on the coast or along rivers) is often cheaper than trade with the hinterland, especially in many developing countries equipped with a fairly poor network of secondary roads. Furthermore, a larger share of urban output is made-up of non-tradable consumer services (restaurants, cinemas, theaters,...), which can only be acquired when living in the city.

[9] The first assumption is made for analytical convenience. The key element is that urban costs rise with city size, a property that is also encountered in non-monocentric city models like Fujita and Ogawa (1982), Lucas and Rossi-Hansberg (2002) or Rossi-Hansberg *et al.* (forthcoming); for a survey, see Mori (forthcoming). The second assumption is made without loss of generality since we can always redefine 'regions' so that there is, indeed, only one city or none in each of them.

includes both commuting and housing costs, and where $\theta > 0$ is a parameter positively related to commuting costs. Note that such an expression for urban costs generally prevails in monocentric city models with fixed lot size, linear commuting costs and an equal redistribution of aggregate land rent (Alonso, 1964; Fujita, 1989).[10] In sum, becoming an urban dweller involves two types of cost: the urban costs proper, namely θH_l, and the entry/education cost, namely f^E. Let Π_l denote the entrepreneurial profit in l. The budget constraint is then given by

$$\kappa^i \left[\int_{\mathcal{V}_l} p_l(\nu) d_l(\nu) \mathrm{d}\nu + \theta H_l + f^E \right] + d_l^0 = w_l^i + \overline{d}^0, \tag{2}$$

where $w_l^i = w_l = 1$ if $i = U$, and $w_l^i = \Pi_l$ if $i = E$. In the latter case, her income (the entrepreneur's profit) also depends on her ability, as will be made clear below.

Maximising (1) subject to (2), the indirect utility of a type-i agent in l can be expressed as $V_l^i = w_l^i + \kappa^i \mathrm{CS}_l + \overline{d}^0$, where CS_l denotes the consumer surplus (see Appendix A.2). Letting $p_{hl}(\nu)$ stand for the price of variety ν produced in h and sold in l, the latter is given by:

$$\begin{aligned} \mathrm{CS}_l \;=\; & \frac{\alpha^2 N_l}{2(\gamma + \eta N_l)} - \frac{\alpha}{\gamma + \eta N_l} \sum_h \int_{\mathcal{V}_{hl}^+} p_{hl}(\nu) \mathrm{d}\nu \\ & + \frac{1}{2\gamma} \sum_h \int_{\mathcal{V}_{hl}^+} p_{hl}^2(\nu) \mathrm{d}\nu - \frac{\eta}{2\gamma(\gamma + \eta N_l)} \left[\sum_h \int_{\mathcal{V}_{hl}^+} p_{hl}(\nu) \mathrm{d}\nu \right]^2 . \end{aligned} \tag{3}$$

As shown by Melitz and Ottaviano (2008), preferences exhibit a taste for variety ($\partial \mathrm{CS}_l / \partial N_l > 0$) and the consumer surplus is decreasing in the average price. The consumer surplus is also increasing in price dispersion, as consumers can reallocate their expenditure towards cheaper varieties (for a given average consumer price, CS_l is increasing in the variance of prices).

2.3 Production

We assume that markets are segmented and that entrepreneurs are free to price-discriminate.[11] The *delivered cost* in city h of a unit produced with marginal cost c in city l is $\tau_{lh} c$, with $\tau_{lh} \geq 1$ (with strict inequality if $h \neq l$). Hence, $(\tau_{lh} - 1)c$ may be interpreted as the frictional trade cost incurred in transporting a unit of any variety of the differentiated good across the two cities. We interpret such a cost in a broad sense as stemming from all distance-related barriers to the exchange of goods.

The variable production component requires using the numéraire good as an intermediate

[10] Appendix A.1 provides micro-foundations for our reduced form of urban costs.

[11] Price discrimination is a prevalent feature across countries. In addition, using barcode microdata, Broda and Weinstein (2008, pp.2-3) find that "the [law of one price] is also flagrantly violated across cities in the same country. Thus, [. . .] prices vary substantially across space even within borders."

input so that the cost function of an entrepreneur with ability c in l is given by

$$C_l\left(\{q_{lh}\}_{h=1}^{\Lambda}, c\right) = c\left(\sum_{h=1}^{\Lambda} \tau_{lh} q_{lh}\right),$$

where q_{lh} is output produced in l and sold in h. Recall that the numéraire good is ubiquitous, i.e., it is available everywhere at the same unit cost since it can be shipped freely. Variable costs are, of course, strictly positive only if the entrepreneur choses to produce.

2.4 Parameterisation and symmetry

To obtain clear analytical results, we henceforth assume that productivity draws $1/c$ in region l follow a Pareto distribution with lower productivity bound $1/c_{l,\max}$ and shape parameter $k \geq 1$. This implies a distribution of cost draws given by:[12]

$$G_l(c) = \left(\frac{c}{c_{l,\max}}\right)^k, \quad c \in [0, c_{l,\max}].$$

The shape parameter k is related to the dispersion of cost draws and is assumed to be the same in all regions. When $k = 1$, the cost distribution is uniform on $[0, c_{l,\max}]$. As k increases, the relative number of low productivity firms increases, and the productivity distribution is more concentrated at these low productivity levels. Any truncation of the Pareto distribution from above at $c^* < c_{l,\max}$ is also a Pareto distribution with shape parameter k. To avoid a taxonomy of special cases, we impose $\alpha < c_{l,\max}$ for all $l = 1, 2, \ldots, \Lambda$ in what follows. This assumption implies that a stand-alone firm that gets a really bad draw will not be productive enough to produce at equilibrium.

In what follows, unless otherwise specified, we further simplify the analysis by imposing some symmetry assumptions across regions. In particular, we assume that the ability distributions of entrepreneurs are identical ($G_l(\cdot) \equiv G(\cdot)$ or $c_{l,\max} = c_{\max}$ for all l). We also assume that trade costs are symmetric and that the costs for trading in the local market are negligible, i.e. $\tau_{ll} = 1$ for all l, which is a normalization:

$$\tau_{lh} = \tau_{hl} = \begin{cases} 1, & l = h \\ \tau > 1, & l \neq h \end{cases}$$

These assumptions may appear restrictive but we impose them for two reasons. First, they greatly ease the analysis and the notational burden without significantly modifying our main

[12] This specific parametrisation for the distribution of productivity draws (entrepreneurs' abilities) leads to a size distribution of establishements which reasonably approximates the empirical one (Axtell, 2001; Bernard *et al.*, 2003; Del Gatto *et al.*, 2006). Actually, the size distribution of firms is approximately lognormal. Using Portuguese data, Cabral and Mata (2003) find that the size distribution of a cohort of firms is very skewed to the right and converges over time to a lognormal distribution. The distribution of the population is also skewed to the right, and the Pareto distribution is a unimodal right-skewed distribution.

theoretical insights. As shown by Tabuchi *et al.* (2005) and by Tabuchi and Thisse (2008), the general case of an urban hierarchy leads to a complex taxonomy and only allows for clear-cut results in a few special cases (involving some symmetry). Second, making these assumptions will emphasize that our model is rich enough to generate various urban configurations *despite all cities sharing the same fundamentals*. However, when taking some key predictions of the model to data, we will rely on a more general specification with asymmetric trade cost.[13]

2.5 Market outcome

Each entrepreneur maximizes operating profits in all markets with prices being her strategies (since firms are atomistic, price and quantity competition are equivalent). Let $p_{hl}(c)$ and $q_{hl}(c)$ denote the price and the quantity sold by an entrepreneur with marginal cost c, when she produces in region h and serves region l. Since markets are segmented and unit input requirements are constant, entrepreneurs independently maximize the operating profits earned from sales to different regions. Let $\pi_{hl}(c) = [p_{hl}(c) - \tau_{hl}c]\, q_{hl}(c)$ denote these operating profits, expressed as a function of the firm's marginal cost c.

Each firm sets profit-maximising prices, taking the other firms' equilibrium strategies as given. An equilibrium may thus be described by: a pricing strategy $p_{hl}(c)$, i.e., a mapping $\{p_{hl}(\cdot)\}_{l=1}^{\Lambda} : \mathbb{R}_+ \to \mathbb{R}_+^{\Lambda}$; an equilibrium mass N_l of entrepreneurs selling to region l; and finally Λ 'enter-or-exit' decisions $\{\mathrm{I}_{hl}(\cdot)\}_{l=1}^{\Lambda} : \mathbb{R}_+ \to \{0,1\}^{\Lambda}$ for each entrant which depends on the marginal cost c. We show in Appendix A.3 that only the most efficient firms make non-negative profits, whereas the least productive ones chose to exit (Lucas, 1978). More precisely, only entrepreneurs with c 'sufficiently smaller' than some cost cutoff c_l are productive enough to sell in city l. The resulting Nash equilibrium prices can then be expressed as follows:

$$p_{hl}(c) = \frac{c_l + \tau c}{2}, \quad \text{where} \quad c_l \equiv \frac{2\alpha\gamma + \eta N_l \overline{c}_l}{2\gamma + \eta N_l} \quad \text{and} \quad \overline{c}_l = \frac{k}{1+k} c_l \tag{4}$$

denote the *domestic cost cutoff in region* l and the average marginal cost of surviving entrepreneurs, respectively. Note the incomplete pass-through of own marginal cost and transportation cost to consumers. Also, the consumer price is decreasing in the degree of competition in the destination market, which is inversely related to c_l (see (6) below). For each pair of cities l and h, there exists an *export cost cutoff* c_{lh} such that only entrepreneurs with c lower than c_{lh} export from l to h. This cutoff must satisfy the zero-profit cutoff condition $c_{hl} = \sup\{c \mid \pi_{hl}(c) > 0\}$. This condition can be expressed as either $p_{hl}(c_{hl}) = \tau c_{hl}$ or $q_{hl}(c_{hl}) = 0$, which from (4) yields:

$$c_{hl} = \frac{c_l}{\tau}. \tag{5}$$

Equation (5) implies that $c_{hl} \leq c_l$ since $\tau \geq 1$. Put differently, trade barriers make it harder for exporters to break even relative to their local competitors because of higher market access costs.

[13] The analytical expressions of the equilibrium conditions in the asymmetric case are given in Appendix A.5.

The mass of entrepreneurs selling in region l is given as follows:

$$N_l \equiv \sum_h H_h G(c_{hl}) = \frac{2\gamma(1+k)(\alpha - c_l)}{\eta c_l}. \tag{6}$$

Note that (6) establishes a positive equilibrium relationship between the number of competitors selling in city l and the toughness of selection: only the entrepreneurs with a productivity $1/c$ larger than $1/c_l$ survive. *The larger the number of competitors, the smaller the fraction* $G(c_l)H_l$ *of entrepreneurs that are fit enough to survive.* Accordingly, we refer to $1 - G(c_l)$ as the 'failure rate' in the urban market. Substituting (4) and (6) into (3), the consumer surplus can finally be expressed very compactly as follows:

$$\text{CS}_l \equiv \text{CS}(c_l) = \frac{\alpha - c_l}{2\eta}\left(\alpha - \frac{1+k}{2+k}c_l\right). \tag{7}$$

Thus, c_l is a sufficient statistic to analyse the impact of any policy or parameter change on consumer welfare. Specifically, as shown by Melitz and Ottaviano (2008), consumer well-being is increasing in the toughness of competition, i.e., $\partial \text{CS}_l / \partial c_l < 0$.

2.6 Equilibrium

It is readily verified that

$$\begin{aligned} \Pi_l(c) &\equiv \sum_h \pi_{lh}(c) = \sum_h \left[p_{lh}(c) - \tau c\right] q_{lh}(c) \\ &= \text{I}_{ll}(c)\frac{H_l}{4\gamma}(c_l - c)^2 + \sum_{h \neq l} \text{I}_{lh}(c)\frac{H_h}{4\gamma}(c_h - \tau c)^2, \end{aligned} \tag{8}$$

where $\text{I}_{lh}(c) = 1$ if $c < c_{lh}$ and $\text{I}_{lh}(c) = 0$ otherwise. In words, $\text{I}_{lh}(c) \in \{0, 1\}$ indicates the mapping that selects firms with productivity $1/c$ as a function of their region (l) and of the destination market (h). Clearly, we have $[0, c_{lh}] = \{c \mid \text{I}_{lh}(c) = 1\}$. The 'utility-gap' for a worker with entrepreneurial ability c between remaining unskilled in the country-side or becoming an entrepreneur in region l is given by:[14]

$$\Delta V_l(c) = \Pi_l(c) + \text{CS}_l - f^E - \theta H_l. \tag{9}$$

Using (7) and (8), expression (9) can be rewritten as follows:

$$\begin{aligned} \Delta V_l(c) &= \text{I}_{ll}(c)\frac{H_l}{4\gamma}(c_l - c)^2 + \sum_{h \neq l} \text{I}_{lh}(c)\frac{H_h}{4\gamma}(c_h - \tau_{hl} c)^2 \\ &\quad + \frac{\alpha - c_l}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_l\right] - f^E - \theta H_l. \end{aligned} \tag{10}$$

[14] Recall that f^E includes the opportunity cost of becoming an entrepreneur, i.e., the wage $w_l = 1$ in the homogenous sector.

A worker decides to become an urban entrepreneur if her expected indirect utility is larger than the (certain) equivalent that she could secure in the numéraire sector. Formally, this is so when $\mathbb{E}(\Delta V_l) \geq 0$. Hence, $\mathbb{E}(\Delta V_l) \leq 0$ must hold at equilibrium, which we henceforth refer to as the *free-entry condition*.

We define a *short-run equilibrium* as a situation in which, contingent on entry decisions summarised by the Λ-dimensional vector $\{H_l\}_{l=1}^{\Lambda}$, (i) entrepreneurs decide to produce or not and how much so as to maximise profits, and (ii) consumers maximise utility. Thus, at any short-run equilibrium, the masses of sellers must obey (6), which we can rewrite as:

$$\frac{\alpha - c_l}{c_l^{1+k} A\eta} \equiv H_l + \tau^{-k} \sum_{h \neq l} H_h \tag{11}$$

where $A \equiv 1/[2c_{\max}^k \gamma(1+k)]$ is a recurrent bundle of parameters. Note that A captures the underlying productivity of the economy as it is decreasing in the upper bound $c_{\max}$ of the support of $G(\cdot)$. Prices adjust quickly but entry decisions are more lumpy. We thus define a *long-run equilibrium* (an *equilibrium* for short) as a 2Λ-tuple $(\{H_l, c_l\}_{l=1}^{\Lambda})$ such that the free-entry and the short-run equilibrium conditions hold simultaneously. In other words, (i) entrepreneurs maximise profits, (ii) consumers maximise utility, and (iii) agents decide whether to become an urban entrepreneur or to stay put as a rural worker. Expected profits, net of urban and entry costs, are non-positive at equilibrium. As shown in Appendix A.4, the expected value of (10) is given by:

$$\begin{aligned} \mathbb{E}(\Delta V_l) &= A\frac{H_l c_l^{2+k} + \tau^{-k} \sum_{h \neq l} H_h c_h^{2+k}}{2+k} \\ &\quad + \frac{\alpha - c_l}{2\eta}\left[\alpha - \frac{1+k}{2+k} c_l\right] - f^E - \theta H_l. \end{aligned} \tag{12}$$

Expectations are rational and, at equilibrium, perfect. Agents are atomistic, hence they rationally disregard the impact of their actions on equilibrium market aggregates; they also take all other agents' decisions as given. Conditions (11) and the free-entry conditions $\mathbb{E}(\Delta V_l) \leq 0$, where $\mathbb{E}(\Delta V_l)$ is from (12), constitute a system of 2Λ equations in the 2Λ unknowns $\{H_l\}_{l=1}^{\Lambda}$ (city sizes) and $\{c_{ll}\}_{l=1}^{\Lambda}$ (cost cutoffs).

3 Equilibrium with one region: 'Urbanisation'

To set the stage, we start by analysing the equilibrium in the model with a single region. This way, we are able to identify the three-way relationship among urbanisation, selection, and polarisation in a parsimonious way. As we shall see, two types of equilibria may arise in this simple case: an equilibrium in which no city forms and an equilibrium in which a city forms. To ease notation, we drop the h and l subscripts for the time being, except for the cutoff (which may otherwise be mixed up with the firms' individual c). Using (12), the free entry condition reduces to

$$\frac{A}{2+k} H c_l^{k+2} + \frac{\alpha - c_l}{2\eta} \left[\alpha - \frac{1+k}{2+k} c_l \right] - f^E - \theta H \leq 0, \tag{13}$$

with equality if $H > 0$ and strict inequality if $H = 0$. Condition (11), which determines the mass of sellers, can be solved for H as follows:

$$H = \frac{1}{A\eta} \frac{\alpha - c_l}{c_l^{1+k}}. \tag{14}$$

Two aspects of (14) are noteworthy. First, at any equilibrium with a strictly positive mass of entrepreneurs ($H^* > 0$), the equilibrium cutoff is strictly smaller than α. Second, $\partial H / \partial c_l < 0$ and $\partial^2 H / \partial c_l^2 > 0$, which shows that at any equilibrium larger agglomerations have lower cost cutoffs than smaller agglomerations, i.e., there is a positive (and convex) equilibrium relationship between *agglomeration* and *selection*. In plain English, a large urban place provides a tougher, more competitive environment than a smaller one, and at rate increasing in city size; as a result, only the fittest entrepreneurs survive and produce, and this effect is particularly strong in large cities. Substituting (14) into (13), and rearranging, we obtain:

$$\frac{\alpha - c_l}{2\eta} \left[\alpha - \frac{k-1}{2+k} c_l - \frac{2\theta}{A c_l^{1+k}} \right] - f^E \equiv f(c_l) \leq 0. \tag{15}$$

The nature and number of equilibria is thus fully characterised by the properties of $f(\cdot)$, which we thus decribe next. An interior equilibrium with a city ($H^* > 0$ and $0 < c_l^* < \alpha$, which we henceforth refer to as an *urban equilibrium*), is such that $f(c_l^*) = 0$; whereas an equilibrium without city ($H^* = 0$ and $c_l^* = \alpha$, which we henceforth refer to as a *rural equilibrium*), necessarily implies that $f(\alpha) \leq 0$. A rural equilibrium is always stable whenever it exists, whereas an urban equilibrium is locally stable if and only if $\partial f(c_l^*) / \partial c_l > 0$. This latter condition implies that, at a locally stable equilibrium, any small perturbation of city size is such that the free-entry condition will bring the economy back to its initial situation.[15]

It is readily verified that $\lim_{c_l \to 0} f(c_l) = -\infty$, which shows that there quite naturally always exists an upper limit to city size. Furthermore, by continuity, whenever a rural equilibrium does not exist there exists at least one stable equilibrium with $0 < c_l^* < \alpha$ (Ginsburgh *et al.*, 1985). A by-product of the last property is that the smallest root of $f(\cdot)$ (whenever one exists), which corresponds to the largest equilibrium city size, is a stable equilibrium as in Henderson (1974). Turning to the structure of equilibria, we can prove the following results.

Proposition 1 (existence and number of equilibria) *The function $f(\cdot)$ has either one or three positive roots, of which at most two are in $[0, \alpha)$. Consequently, there exist at most two*

[15] This is, e.g., the case if, following the standard NEG tradition, we specify the following law of motion for H_l: $\dot{H}_l = \mathbb{E}(\Delta V_l) H_l (L_l - H_l)$. Such an ad-hoc law of motion can be micro-founded (see Baldwin, 2001).

stable equilibria: the urban equilibrium and the rural equilibrium. If no stable urban equilibrium exists, then the rural equilibrium is unique. Furthermore, the equilibrium associated with the smallest value of c_l (the largest H) is always stable.

Proof. See Appendix B.1. ■

Insert Figures 1(a)–1(c) about here.

As shown by Proposition 1, only three equilibrium configurations may occur, which are depicted by Figures 1(a)–1(c). The actual outcome depends on the parameter values of the model. Figure 1(a) illustrates the situation in which none of the roots of $f(\cdot)$ belongs to the relevant range $[0, \alpha]$.[16] In that case, $f(\alpha) < 0$ and nobody enters the city. Turn next to Figure 1(b), which illustrates the case in which $f(\cdot)$ admits a unique root in $[0, \alpha]$, denoted by c_L^0 ('L' for 'low').[17] In that case, $f(\alpha) > 0$ so that some agents always have an incentive to enter the city. The rural equilibrium does not exist for such parameter configurations. Finally, consider Figure 1(c). Here, $f(\cdot)$ admits a second root in $[0, \alpha]$, denoted by c_M^0 ('M' for 'middle'). The smallest root alone corresponds to a stable (urban) equilibrium. In that case, $f(\alpha) < 0$ and the rural equilibrium co-exists with an urban equilibrium.[18]

Given the equilibrium structure, how do the equilibria change with the value of the underlying parameters? Although $f(\cdot)$ is a transcendental function of c_l and, therefore, does not allow for algebraic expressions of its roots, we establish the following clear comparative static results.

Proposition 2 (monotonicity of urban equilibria) *All stable equilibrium city sizes H^* are non-increasing in θ and non-decreasing in A and α. Put differently, lower commuting costs (lower θ), a better productivity support (lower $c_{\max}$ and thus higher A), more product differentiation (lower γ and thus higher A), and stronger preference for the differentiated good (larger α) all weakly increase city size at any stable equilibrium.*

Proof. See Appendix B.2. ■

Proposition 2 establishes that any improvement in the benefits of living in cities, either as consumers or entrepreneurs, makes the emergence of cities more likely and maps into larger equilibrium city sizes. By the same token, any reduction in urban costs that stems from an improvement in urban transportation, say, is conducive to urban growth (Duranton and Turner, 2008). These results are consistent with the historical fact that the Industrial Revolution triggered a massive urbanisation process which led to the current, unprecendented urbanisation rates: the U.S., which is currently the most urbanised large economy, had an urbanisation rate of only 6% in

[16] Three sub-cases may arise, depending on the shape of f, but in all of them $f(c_l) < 0, \forall c_l \in [0, \alpha]$.

[17] Three sub-cases may arise, depending on the shape of f, but in all of them $f(c_L^0) = 0$.

[18] A third root exists in this case, c_H^0 ('H' for 'high'), but it can be shown that $\alpha \leq c_H^0$. See Appendix B.1.

1800, whereas nowadays more than 80% of its population lives in cities. Our results also support the view that supply and demand factors are crucial in giving rise to and sustaining large metropolitan areas. Specifically, at least three conditions must be satisfied for cities to emerge and develop (see, e.g., Bairoch, 1988; O'Sullivan, 2007). First, there must be an agricultural surplus so that the rural population may feed the urban dwellers (in our model, this condition is trivially satisfied by the initial endowment in the numéraire $\overline{d}^0$). Conversely, there must be some demand for urban goods and services; in the model, the extent of this demand is captured by the parameter α. Also, urban production is more valuable if product differentiation γ is large. Second, the urban population must supply goods and services to sustain itself.[19] It is able to produce more, the more productive it is, i.e., the lower is $c_{\max}$. Last, the transport system must be efficient enough so that commuting is feasible even as city sizes increase. To sum up, a large α or γ and a low θ or $c_{\max}$ are all conducive to the emergence of large cities.

When does which type of equilibrium arise? Closer examination of condition (15) allows us to establish the following results.

Proposition 3 (rural equilibria) *The rural equilibrium ($H^* = 0$ and $c_l^* = \alpha$) exists and is stable for all $f^E > 0$. When $f^E = 0$, it still exists but is a stable equilibrium if and only if $\theta \geq \theta^U \equiv A\alpha^{2+k}/(2+k)$.*

Proof. Condition (14) implies that $H = 0$ if and only if $c_l = \alpha$. Plugging this result into (15) shows that it always holds for any $f^E > 0$. Local stability of the rural equilibrium then immediately follows from the strict inequality. When $f^E = 0$, local stability of the rural equilibrium requires that $\partial f(\cdot)/\partial c_l > 0$ when evaluated at $\{H^*, c_l^*\} = \{0, \alpha\}$. Some straightforward computations and rearrangements, using (15), show that this is equivalent to $\theta > \theta^U$, where θ^U is given by

$$\theta^U \equiv A\frac{\alpha^{2+k}}{2+k}. \tag{16}$$

This establishes our result. ■

Proposition 3 reveals that the rural equilibrium is stable for all strictly positive values of f^E. The intuition is that since there is no urban market when nobody has established a city yet, no one can profitably enter individually (as she has to pay some strictly positive net entry cost f^E). In addition, as also shown by Proposition 3, even when there are no net entry costs for becoming an entrepreneur, the rural equilibrium exists and may be stable provided that either urban costs (θ) are large, or the underlying productivity (A) is low, or preferences for the differentiated good (α) are weak, or product differentiation ($1/\gamma$) is low. Conversely, when θ is low enough ($\theta < \theta^U$), urbanisation necessarily occurs (hence the superscript 'U'). Hence θ^U denotes an urbanisation

[19] In an extension of the model in which cities trade with the countryside, urban dwellers exchange urban products and services to purchase the agricultural surplus; the fraction of their output they so give away is smaller the larger is their productivity.

threshold. When the net entry cost into entrepreneurship is zero ($f^E = 0$), expression (15) is (by virtue of $\alpha > c_l$) equivalent to:

$$\frac{2\theta}{A} \geq c_l^{1+k} \left(\alpha - \frac{k-1}{2+k} c_l \right), \tag{17}$$

the right-hand side of which is strictly concave in c_l, increasing when $c_l \to 0$, and its maximum value is given by $3\alpha^{k+2}/(k+2)$. The roots of equation (17) cannot be generally solved for. However, the condition

$$\theta > \theta^R \equiv \frac{3A\alpha^{k+2}}{2(2+k)} = \frac{3}{2}\theta^U$$

is sufficient to ensure that there exists no pair $\{H, c_l\}$ with $c_l \in (0, \alpha)$ and $H = H(c_l)$ from (14) that is compatible with an equilibrium. In other words, when θ is large enough ($\theta > \theta^R$), the economy remains necessarily rural (hence the superscript 'R'). The following proposition summarises these findings:

Proposition 4 (equilibrium structure when $f^E = 0$) *Assume that there are no net entry costs for becoming an entrepreneur ($f^E = 0$). Then: (i) $H^* = 0$ is the only stable equilibrium for all $\theta > \theta^R$; (ii) there exists a $\{H^*, c_l^*\}$ in $\mathbb{R}_{++} \times (0, \alpha)$ that is the unique stable equilibrium for all $\theta < \theta^U$; and (iii) for $\theta^U < \theta < \theta^R$, both $H^* = 0$ and some $\{H^*, c_l^*\}$ in $\mathbb{R}_{++} \times (0, \alpha)$ are stable equilibria.*

Proof. Parts (i) and (ii) are straightforward applications of the previous results. Part (iii) is derived in Appendix B.3, which establishes that $\theta < \theta^R$ is is also sufficient for an urban equilibrium to co-exist with a rural one. ■

Figure 2 illustrates the equilibrium configurations in this case by plotting equilibrium city sizes against the unit distance communiting cost. Stable equilibria appear in plain curves and unstable equilibria are dashed.

Insert Figure 2 about here.

To summarise our foregoing findings, we think about θ^U as being the urbanisation threshold: for all θ smaller than θ^U, urbanisation *must* occur. Conversely, the *rural threshold* θ^R is an 'urban underdevelopment point' since for all values of θ larger than θ^R no city can emerge. In between, both an urban and a rural equilibrium can be sustained. Because the urbanisation threshold lies below the rural threshold, there is path dependency in the economy. Starting from high commuting costs, those costs must fall sufficiently for cities to emerge. However, such costs can rise to higher levels later again without making the city disappear. This feature of our model captures the resilience of big cities that nowadays feature high monetary and time costs of commuting and congestion.[20] Condition (17) allows for a full comparative static analysis of equilibrium in the simple case where $f^E = 0$. For example, a larger θ, a larger $c_{\max}$ (and hence a smaller A) or a larger γ all increase c_l and, thus, decrease H (see Proposition 2). When the commuting technology deteriorates (or when traffic congestion increases), or when entrepreneurs draw from a worse ability support, or when products are less differentiated, aggregate productivity falls and the equilibrium city size shrinks as entry becomes less profitable.

3.1 Entrepreneur city vs consumer city

Viewing the history of urbanisation following the Industrial Revolution as an ongoing reduction in commuting costs θ, we observe that the early cities are rather small and not very productive ($\{H^*, c_l^*\} \simeq \{0, \alpha\}$ when $\theta \simeq \theta^U$). In such economies, prospective entrepreneurs migrate to cities predominantly because they expect to reap positive profits. Urban migration is primarily motivated by wages that are large relative to rural wages. Urban nominal wages can be relatively high because the concentration of production and demand gives rise to increasing returns to scale that outperform domestic or artisanal production. Put differently, the small initial cities are mainly *production cities*. Furthermore, the 'failure rate' $1 - G(c_l) \simeq 1 - G(\alpha)$ is relatively low, i.e., the mass of unsuccessful entrepreneurs is small. However, the consumer surplus is rather small too in this case: $\text{CS}(c_l) \simeq \text{CS}(\alpha) = 0$ by (7). In sum, urban costs are compensated for by (expected) positive profits in the early modern history of urbanisation. Conversely, when θ is sufficiently low so that urban markets become large and competitive, the failure rate $1 - G(c_l)$ is much larger. Expected profits are then no longer the primary driver of urban life as the city's local and specific service and product mixes work like local amenities to attract consumers that display preference for diversity (Lösch, 1940; Brueckner *et al.*, 1999). At the limit, when $c_l \to 0$, expected profits go to zero and the consumer surplus $\text{CS}(0)$ reaches its maximum and compensates for urban costs on its own. In other words, large modern cities are predominantly *consumer cities* (Glaeser *et al.*, 2001).[21]

It is also worth stressing that, after entry, unsuccesful entrepreneurs in the city have lower nominal and real incomes than those in the country-side, yet their consumer surplus exceeds that in the country-side. The reason is that they have access to urban diversity even if their choice to move to the city turns out to be unsuccessful (recall that unsuccessful agents can still pay

[20] For example, according to the Daitoshi kotsu census (Major Cities Traffic Census) of the Ministry of Land, Infrastructure and Transport in Japan, the average commuting times in 2000 for a one-way trip to work were of about 67, 59 and 61 minutes for Tokyo MA, Osaka MA, and Nagoya MA, respectively. For the U.S., according to Forbes: "the average commuter [in Houston] spends 20.9% of his annual household costs on getting to work." (http://www.forbes.com/2007/08/07/commute-housing-expensive-forbeslife-cx_mw_0807realestate.html)

[21] The terminology 'producer and consumer cities' was introduced by Weber (1958), though his concept of 'consumer city' mirrors more closely the predatory behaviour of primate cities (Ades and Glaeser, 1995) than the more modern concept of consumer city by Glaeser *et al.* (2001).

for urban goods using their initial endowment). This aspect is taken into account in the entry decision in our model. It is reminiscent of standard arguments for explaining the growth of cities in the Third World, where the massive urbanisation in the face of urban poverty constitutes a classical puzzle (see, e.g., Harris and Todaro, 1970).

3.2 Survival of the fittest and polarisation

From expression (14) we know that, at equilibrium, large cities are more productive. But are they also more unequal? This question is warranted since not all entrants produce in equilibrium. Only $G(c_l)H$ entrepreneurs remain in the market, whereas the remaining do not produce and consume from their endowments. The failure rate $1 - G(c_l)$ in the urban market influences the distribution of income across successful and unsuccessful entrepreneurs.

Our model allows us to take a new theoretical perspective on this question. To do so, we first compute the average (operating) profit of all entrants as follows: $\overline{\Pi} = AHc_l^{2+k}/(2+k)$. Making use of the equilibrium relationship (14) between size and productivity, we then obtain: $\partial\overline{\Pi}/\partial c_l = (\alpha - 2c_l)/[\eta(2+k)]$, which is $\cap$-shaped. Average profits first increase in c_l for $0 \le c_l \le \alpha/2$ and then decrease for $\alpha/2 \le c_l \le \alpha$. Note that average profits include those of entrants who fail to succeed in the city. It is hence informative to also compute the average profit conditional upon survival, which is given by $\overline{\Pi}\mid_{c\le c_l} = Hc_l^2/[2\gamma(1+k)(2+k)]$. Some standard calculations, using again (14), yield

$$\frac{\partial}{\partial c_l}\left(\overline{\Pi}\mid_{c\le c_l}\right) = -\left(\frac{c_l}{c_{\max}}\right)^{-k}\frac{c_l + (\alpha - c_l)(k-1)}{\eta(2+k)} < 0,$$

where the inequality is due to $0 \le c_l \le \alpha$ and $k \ge 1$. Hence, the average profit conditional upon survival is increasing (and convex) in city size. The foregoing results show that entrepreneurs who succeed reap higher average profits in larger cities, whereas the overall average (including those who fail) decreases in city size beyond some threshold. Hence, larger cities are conducive to the appearance of 'superstars' at the top of the urban income distribution (Rosen, 1981).

Turning to the variance of incomes ($\sigma^2 \equiv \int \Pi^2(c)\mathrm{d}G(c) - \overline{\Pi}^2$), some longer computations show that it is given by

$$\sigma^2 = c_{\max}^{-k}\left[\frac{H_l}{2\gamma(1+k)(2+k)}\right]^2 c_l^{4+k}\left[\frac{6(1+k)(2+k)}{(3+k)(4+k)} - \left(\frac{c_l}{c_{\max}}\right)^k\right].$$

One can show that σ^2 is first increasing and then decreasing in c_l. Put differently, starting from small and unproductive cities ($c_l \approx \alpha$), the model says that income polarization (as measured by the variance of incomes) first increases and then decreases with size as selection gets tougher. To understand this result, we can decompose the variance between successful entrepreneurs, on the one hand, and those who fail, on the other. All those who fail get the same outcome: the variance conditional upon failing is zero. The variance conditional upon survival is given by

$$\sigma^2 \Big|_{c \leq c_l} = c_l^4 \left[\frac{c_{\max}^k}{\eta(2+k)} \frac{\alpha - c_l}{c_l^{1+k}} \right]^2 \frac{k(11+5k)}{(1+k)(2+k)(3+k)(4+k)}.$$

It is readily verified that this is decreasing in c_l (hence increasing in city size). In words, the variance in profits of successful entrepreneurs is actually increasing in city size, i.e., there is more inequality among the successful in larger cities. The foregoing results show why the overall variance is not monotone: the 'failure rate' increases in larger cities, so that the variance goes down as the group of those who fail are 'uniformely poor' (they all get zero operating profit, or $-f^E$ in total). This trend is not offset by the increasing variance for the successful, as the latter become a smaller share of the total. Eventually, the variance-reducing 'between' effect takes over the variance-enhancing 'within' effect.

Although the variance is decomposable, it is not the best measure of inequality since it is scale dependent. Let us then compute the Gini coefficient of the income distribution in city l (see Appendix B.4 for details):

$$\text{Gini}_l(k; c_l) = 1 - \frac{2+k}{2+4k} \left(\frac{c_l}{c_{\max}} \right)^k. \tag{18}$$

Note that this coefficient does not directly depend on city size H, because the Gini coefficient is 'scale free'. However, size matters indirectly since we know that, in equilibrium, c_l is a decreasing function of city size. Straightforward inspection of (18) yields the following results:

Proposition 5 (polarization) *Let income inequality be measured by the Gini coefficient. Then income inequality is: (i) increasing at an increasing rate in the productivity cutoff* $1/c_l$*; (ii) increasing at a decreasing rate in city size* H_l*; and (iii) increasing in* $c_{\max}$.

Proof. (i) It is readily verified that $\partial(\text{Gini}_l)/\partial c_l < 0$ and $\partial^2(\text{Gini}_l)/\partial c_l^2 < 0$. (ii) Invert (18) to get an expression of c_l as a function of Gini_l, and substitute this for c_l into (14). Then, standard algebra reveals that $\partial^2 H_l/\partial(\text{Gini}_l)^2 > 0$ and thus $\partial^2(\text{Gini}_l)/\partial H_l^2 < 0$. (iii) This part of the proposition is immediate by inspection of (18). ■

To check whether such relationships between size, productivity and inequality are borne out in the data, and to revisit the issue of city size and income inequality, we use data from the 2006 American Community Survey, which provides data for 507 Core Based Statistical Areas (henceforth, CBSA).[22]. Our three key variables are the total population size of the CBSA in millions (size), the median household income of the CBSA in thousand US$ (medi), and the household income Gini coefficient of the CBSA (gini). The partial correlations among these three variables are consistent with the idea that larger metropolitan areas generate both more wealth

[22]The data includes 367 metropolitan and 140 micropolitan statistical areas.

and more inequality. Indeed, these correlations are of .321 between median income and CBSA size, .182 between CBSA size and the income Gini, and of −.277 between the median income and the income Gini. Note that these partial correlations are in accord with the predictions of our model but, of course, in no way indicative of any causality.

To get more detailed information on the relationship between size, productivity and inequality, we next regress the Gini coefficient on population size and squared population size, controlling for median household income and various other factors.[23] We control for median income because we know from numerous previous studies that larger urban areas are associated with higher productivity and wages. Thus, we want to isolate the direct effect of city population size from its indirect effect via the (somewhat mechanical) reduction of the Gini coefficient by the median income. We also include the square of population size to capture the non-monotonic effects present in our model. We expect the Gini coefficient to rise with population size (large agglomerations are more unequal), the coefficient on the square of population size to be negative (decreasing marginal effect), and the Gini coefficient to fall with median income.

Insert Table 1 about here.

As can be seen from our basic specificationa (i) and (ii) in Table 1, even after controlling for the fact that larger areas have higher median income (which reduces the Gini), *larger areas are more unequal*. Put differently, larger urban areas generate more wealth and raise median incomes, yet the polarisation effect is stronger and larger areas are associated with more inequality. However, the increase occurs at a decreasing rate, which is in accord both with the decomposition of the variance across groups derived from our model and from our theoretical results on the Gini coefficient. Our baseline results suggest that successful households are better off in larger cities, but that by far not every household succeeds. Our estimates tell us, for example, that increasing the population from 12,950,129 in Los Angeles-Long Beach-Santa Ana MSA to 18,818,536 in New York-Northern New Jersey-Long Island MSA raises the Gini by about .0622 from its baseline value of .481, after controlling for differences in median household income (which is about 6.78% higher in New York than in Los Angeles and which reduces the Gini by .0056). These figures are quite substantial and suggest that the link between size and inequality is a prevalent and quantitatively important phenomenon.

Specifications (iii)—(v) in Table 1 include various controls that have been found to be important in the empirical literature on city size and inequality.[24] As can be seen from (iii), the

[23] A word of caution is in order. Like all the econometric exercises we conduct in this paper, our evidence ought to be considered as purely indicative; we do not claim to have come up with a proper and sophisticated identification strategy. Rather, we show that the partial correlations that exist in the data support the theoretical predictions of our model.

[24] Previous empirical literature in the late 70s and early 80s has found mixed results as to the relationship between city size and income inequality (e.g., Long *et al.*, 1977; Nord, 1980). Most of those studies, however,

share of Black and Afro-Americans in the CBSA has a positive impact on income inequality. Further, as can be seen from (iv), more educated cities are more unequal. Last, as shown by (v), including all the statistically significant controls does not invalidate the existence of a positive and significant relationship between size and inequality. Though the coefficient on size is reduced, it remains significantly positive, whereas size squared still enters negatively. Specification (vi) replaces size with density. As can be seen, we also obtain a positive impact of density on the Gini coefficient. A likely explaination is that density increases productivity and selection (Ciccone and Hall, 1996; Syverson, 2004), thereby generating a more unequal distribution of income. Last, specification (vii) uses the 1-year estimates of the 2007 American Community Survey to replicate the basic estimations. We again obtain a significant relationship between inequality and size, which appears to be almost the same than that in 2006.

To summarize our main findings, *large urban areas generate more wealth and are at the same time more unequal (polarised) than smaller cities.* This theoretical prediction of our model shows up highly robustly in the U.S. CBSA data.

4 Symmetric equilibria: 'Trading cities'

In this section, we let $\Lambda \geq 2$ to look for the existence of symmetric equilibria with multiple regions and to characterise their properties. The analysis is similar to the case with $\Lambda = 1$, except for the existence of trading links across regions. We thus exclusively focus on the impact of changes in trade costs on the existence and the properties of symmetric equilibria. Propositions 1, 2 and 3 from the previous section are readily generalised to this new setting, therefore their formal extensions are relegated to Appendix C to save space.

4.1 Trade and the rise of cities

Let $\Phi \equiv (\Lambda - 1)\tau^{-k}$ denote the 'freeness' of trade (Baldwin *et al.*, 2003). This collection of parameters takes value $\Phi = 0$ when $\tau \to \infty$ (trade is prohibitive), or when $\Lambda = 1$ (there is a single isolated region), and value $\Phi = \Lambda - 1$ when $\tau \to 1$ (trade is costless). It is increasing in Λ and decreasing in τ.[25] Since the model is perfectly symmetric by assumption, an equilibrium where all regions have the same size H_l and the same cutoff c_l always exists. In the symmetric case with trade, the free-entry condition (12) in each region reduces to:

$$\frac{(1+\Phi)A}{2+k} H_l c_l^{2+k} + \frac{\alpha - c_l}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_l\right] - f^E - \theta H_l \leq 0. \tag{19}$$

highlight that larger cities are more unequal than smaller ones in the U.S. Quite surprisingly, this literature seems to have largely disappeared after the mid-1980s.

[25]Note also that a distribution which is more skewed towards lower ability draws (a higher value of k) implies a lower Φ for any given τ, as fewer entrepreneurs are productive enough to export to other cities.

Likewise, (11) becomes

$$H_l = \frac{1}{(1+\Phi)A\eta} \frac{\alpha - c_l}{c_l^{1+k}}. \tag{20}$$

Substituting (20) into (19), and rearranging, we then obtain:

$$\frac{\alpha - c_l}{2\eta} \left[\alpha - \frac{k-1}{2+k} c_l - \frac{2\theta}{(1+\Phi)A c_l^{1+k}} \right] - f^E \equiv f(c_l) \leq 0. \tag{21}$$

Rural and urban equilibria are defined as in the single-region case of Section 3. We can show the following results.

Proposition 6 (cities and trade) *A larger Φ (lower trade costs τ and/or more trading partners Λ), a larger A or a lower θ all make the existence of cities more likely and weakly increase their equilibrium size.*

Proof. Proposition 11 in Appendix C generalises Proposition 1 to the Λ-city symmetric case. It is a technical condition and requires no further discussion. Next, Propositions 12 and 13 in Appendix C together establish that any improvement in the benefits of locating in cities, either as consumers or entrepreneurs, result in larger city size and make the existence of cities more likely, as did Propositions 2 and 3 in the previous section. More formally, a low value of Φ makes the rural equilibrium more likely to occur. Indeed, as shown in Appendix C, if $f^E = 0$ then there exists a unique value (the urbanisation threshold), given by

$$\theta^U \equiv (1+\Phi)A \frac{\alpha^{2+k}}{2+k}$$

such that the rural equilibrium exists and is stable for all $\theta \geq \theta^U$, whereas the urban equilibrium is the unique stable equilibrium when $\theta < \theta^U$.[26] Likewise, there exists a unique $\theta^R \equiv \frac{3}{2}\theta^U$ such that the urban equilibrium exists and is stable for all $\theta < \theta^R$, whereas the rural equilibrium is the unique stable equilibrium when $\theta \geq \theta^R$. Clearly, θ^U and θ^R are increasing in Φ (i.e., with freer trade), which proves our claim. ■

As established before in Propositions 3 and 4, the *intra-city transportation system* must be efficient enough for cities to emerge in equilibrium. The new result in Proposition 6 is that cities are also more likely to emerge if the *inter-city transportation system* is efficient enough so that cities can trade with one another. Note that this result is not as obvious as it may sound. Indeed, from the perspective of entrepreneurs in each city, lower inter-city trade costs and a larger number of trading partners mean both a better market access (Krugman, 1980) and tougher competition from entrepreneurs established in other cities (Ottaviano *et al.*, 2002). The latter effect can be seen from (20), where, given c_l, $\partial H_l / \partial \Phi < 0$. As it turns out, Proposition 6

[26] In this section, we 'reset' notation in the sense that we redefine the urban cost thresholds of the previous section. Obviously, when $\Phi = 0$ the thresholds of the two sections are identical.

shows that in equilibrium the agglomeration effect dominates, i.e., $\partial H_l^*/\partial\Phi > 0$; the selection effect is also stronger, the lower the trade costs τ are and/or the more numerous the trading cities Λ are, i.e., $\partial c_l^*/\partial\Phi < 0$.

Finally, as shown by Proposition 14 in Appendix C, Proposition 3 also straightforwardly generalises to the case of trading cities.

4.2 Trade, profits and polarisation

Does trade contribute to rising profits and larger earning inequalities in cities? To answer this question, we first compute the average (operating) profit of the entrants in the symmetric case as follows:

$$\overline{\Pi} = \frac{(1+\Phi)A}{k+2} H c_l^{2+k} = \frac{c_l(\alpha - c_l)}{\eta(2+k)},$$

where we have used (20). Since $\partial c_l/\partial\tau > 0$, it is readily verified that the average profit in city l is ∩-shaped. Hence, as in the single-city case with respect to size, average profits are first increasing as trade costs fall from high initial values, and then eventually decreasing as trade becomes sufficiently free. In the early stages of integration, access to a larger market raises entrepreneurs' profits, whereas in later stages of integration increased competition reduces them again as more agents fail due to tougher selection. The average profit conditional upon survival is given by

$$\overline{\Pi}\,\Big|_{c\le c_l} = \frac{(1+\Phi)}{2\gamma(1+k)(2+k)} H c_l^2 = \frac{1}{2\gamma(1+k)(2+k)A\eta} \frac{\alpha - c_l}{c_l^{-1+k}}$$

where we have again used (20). Since $\partial c_l/\partial\tau > 0$, average profits conditional upon survival are strictly increasing as trade becomes freer. In words, trade makes entrepreneurs and firms more profitable as it increases average productivity. The rest of the comparative static results is as in the single-city case and one can check that our results go through when we consider the freeness of trade Φ instead of the trade cost τ.

Turning to the variance, the expression is too unwieldy to be revealing. However, the computation of the Gini coefficient yields some clear results. It is now given by

$$\text{Gini}_l(\Lambda,\tau,k;c_l) = 1 - \lambda(\Lambda,\tau,k)\left(\frac{c_l}{c_{\max}}\right)^k, \tag{22}$$

where $\lambda(\cdot)$ is a bundle of parameters, the properties of which are relegated to Appendix C.2. One can verify that $\lambda(1,\tau,k) = (2+k)/(2+4k)$, which reduces to the one-city case (18). Conversely, when inter-city trade is perfectly free, the Gini coefficient (given c_l) is also the same as in the one-city case, i.e. $\lambda(\Lambda,1,k) = (2+k)/(2+4k)$.

Insert Figures 3(a) and 3(b) about here.

Figures 3(a) and 3(b) plot the equilibrium Gini index $\text{Gini}_l(\Lambda, \tau, k)$ as a function of τ for $\Lambda = 4$ and $\Lambda = 10$, respectively, by using the equilibrium value of (21) in (22). All the simulations we have conducted show that inequalities rise as the number of trading partners increases, or $\partial\text{Gini}_l/\partial\Lambda > 0$ (see Appendix C.2). Turning to trade costs τ, we can algebraically show the following result.

Proposition 7 (trade and polarisation) *Let income inequality be measured by the Gini coefficient at the symmetric equilibrium. Then income inequality is decreasing in trade/transportation costs, namely* $\partial\text{Gini}_l(\Lambda, \tau, k)/\partial\tau < 0$.

Proof. By inspection of (22), Gini_l is decreasing in both c_l and $\lambda(\cdot)$. Thus, to establish the result in the Proposition, it is sufficient to show that $\partial\lambda(\Lambda, \tau, k)/\partial\tau < 0$, which we do in Appendix C.2. ■

Proposition 7 shows that trade integration and/or lower costs of shipping goods do not favor more equality in earnings, at least in the symmetric case. The reason is that freer trade makes selection tougher, which raises the incomes of the most productive agents but makes more agents fail. As we show in appendix C.2, a lower τ also redistributes income from the least productive entrepreneurs to the most successful ones, thus lowering $\lambda(\cdot)$ as a result. Consequently, the income distribution gets more skewed towards the most productive agents, who secure a larger share of total income. The recent trends of international integration are hence likely to spur more income inequality at the city level.

The theoretical prediction of Proposition 7 does not open itself easily to cross-section scrutinity, since it holds by construction at the symmetric equilibrium. Instead, assuming that the U.S. system of cities could be reasonably approximated by the symmetric equilibrium of our model (certainly as strong an assumption as it gets), we could use panel data to see whether urban income inequalities are increasing as inter-city transportation and trade costs fall. Unfortunately, such data are not available. We leave to future work the use of individual data to construct such a panel of Gini coefficients and other measures of income inequalities. Nevertheless, we show in Section 5 how market potential (which weights the inverse of bilateral distances by city size) is relevant in explaining how 'centrality' and income inequality interact.

4.3 Stability of symmetric equilibrium

The exhaustive study of the stability properties of the equilibrium in the general case where $\Lambda > 2$ is a daunting task because of the complexity of spatial interdependencies (Fujita *et al.*, 1999a, 1999b; Tabuchi *et al.*, 2005; Tabuchi and Thisse, 2008). It is, therefore, beyond the scope of the main text of this paper.[27] Instead, we study a meaningful special case that allows us to

[27] In the technical Appendix TA.1, we provide additional stability results for the general case. Furthermore, in the technical Appendix TA.2, we present a general numerical procedure that allows to computationally assess the

parsimoniously illustrate the five key ingredients that shape the spatial outcome of our model: selection, competition, agglomeration ('backward' and 'forward' linkages) and urban costs.

In what follows, we retain a simple definition of stability: we consider that the symmetric equilibrium is (locally) stable if an arbitrarily small shock (to the endogenous variables) common to *all* cities is self-correcting, i.e. $\mathrm{d}f(\cdot)/\mathrm{d}c_l > 0$ at the symmetric equilibrium. To be even more specific, we focus solely on the case without aggregate shocks, i.e. $\sum_l \mathrm{d}H_l = 0$, and where the shock affects *just two cities* at the symmetric equilibrium.[28] When is such a shock to city size self-correcting? In other words, starting from a symmetric equilibrium configuration, imagine that H_l increases by $\mathrm{d}H > 0$ and H_h decreases by the same $\mathrm{d}H > 0$, for some $l, h \leq \Lambda$ (and $\mathrm{d}H_i = 0$ for all $i \neq l, h$). Loosely speaking, this corresponds to an entrepreneur entering the 'wrong' city. If $\mathbb{E}[\Delta V_l(\mathrm{d}H)] = -\mathbb{E}[\Delta V_h(\mathrm{d}H)]$ is negative, then the shock is self-correcting and the symmetric equilibrium is indeed locally stable (the symmetry of the model implies $\mathbb{E}[\Delta V_i(\mathrm{d}H)] = 0$ for all $i \neq l, h$). Otherwise, symmetry is 'broken' in the sense of Krugman (1991) and Puga (1999) and the symmetric equilibrium is locally unstable (Baldwin, 2001).

To address this issue formally, let $H_l = H_h = H$, $\mathrm{d}H = \mathrm{d}H_l = -\mathrm{d}H_h > 0$ and $\mathrm{d}c_h = -\mathrm{d}c_l$. Let also $\phi \equiv \tau^{-k}$ to alleviate notation. Differentiating (11) and (12) around the symmetric equilibrium, and using (20), yields

$$\begin{aligned}
\mathrm{d}\mathbb{E}(\Delta V_l) \quad = \quad & \underbrace{-\theta \mathrm{d}H}_{\text{congestion}} + \underbrace{\frac{1-\phi}{1+\Phi}\frac{\alpha - c_l}{\eta}\frac{c_l}{2+k}\frac{\mathrm{d}H}{H}}_{\text{backward linkage}} \\
& + \underbrace{\frac{1-\phi}{1+\Phi}\frac{\alpha - c_l}{\eta}\mathrm{d}c_l}_{\text{selection and competition}} - \underbrace{\frac{1}{\eta(2+k)}\left[\frac{\alpha}{2} + (\alpha - c_l)(1+k)\right]\mathrm{d}c_l}_{\text{forward linkage}} \qquad (23)
\end{aligned}$$

and

$$0 \equiv (1+\Phi)\frac{\alpha + k(\alpha - c_l)}{\alpha - c_l}\frac{\mathrm{d}c_l}{c_l} + (1-\phi)\frac{\mathrm{d}H}{H}. \qquad (24)$$

A few aspects of (23) are noteworthy. First, the expected value of becoming an entrepreneur in l is affected by both the mass of entrepreneurs, H, and their average equilibrium ability (which is proportional to c_l under the Pareto parametrisation). Second, there are a 'pull' and a 'push' factor in both. To see this, consider the first line of the right-hand side of (23). An additional

stability of any equilibrium candidate, including any corner solution.

[28] The absence of aggregate shocks is standard in economic geography models where the mass of firms and of mobile agents is usually fixed (Krugman, 1991; Ottaviano *et al.*, 2002). In the general case, local stability requires that at any interior equilibrium, the Jacobian of $f(\cdot)$ with respect to $\mathbf{c} = (c_1 \ c_2 \ \ldots \ c_\Lambda)$ be positive definite. In the numerical applications presented in Section 5, we check this condition systematically by verifying that all eigenvalues of the Jacobian of $f(\cdot)$ are strictly positive (see the technical Appendix TA.2). However, to highlight the key mechanisms that contribute to symmetry-breaking, we restrict ourselves to a less stringent necessary condition in the remainder of this section. The rationale for doing so is that it is well known that characterising the eigenvalues of a non-numerical system is a formidable task that leads to a complex taxonomy of different cases (see Tabuchi and Zeng, 2004, and Tabuchi *et al.*, 2005, for additional details on stability of spatial equilibria).

entrepreneur in l means one more urban dweller, which increases urban costs by θ (*congestion*); it also means one more consumer in l (and one fewer in h), which means a relatively larger market in l and thus higher profits; this demand linkage is also known as a *backward linkage* (Fujita *et al.*, 1999b). The latter effect relies on market segmentation, hence it is increasing in τ (i.e., decreasing in ϕ and Φ); at the limit, when goods markets are fully integrated ($\phi = 1$), this effect vanishes as local market size becomes irrelevant. Consider next the second line of the right-hand side of (23). Less productive entrepreneurs (i.e., a larger value of c_l) is good news for expected profits: the failure rate is lower (less *selection*) and the pro-competitive effect is weaker (*competition* is softer). This effect is also directly affected by market segmentation: as $\phi \to 1$, competition becomes *global* and thus shifting entrepreneurs around has no impact on *local* expected profits. By contrast, less productive domestic entrepreneurs is bad news for consumers because they pay higher prices for their consumption bundle. The reason is that, since all varieties are substitutes, less productive domestic entrepreneurs raise the prices of all varieties sold in l. Also, consumers substitute towards non-local varieties as c_l rises and pay trade costs on these imports. This cost linkage, also known as *forward linkage* (Fujita *et al.*, 1999b), is indirectly affected by the degree of market segementation. Indeed, as can be seen from (24), the link between c_l and H weakens as ϕ decreases: swapping entrepreneurs between l and h has no effect on consumer surplus nor on competition when the market is global.

5 Asymmetric equilibria: 'Urban systems'

We define as an *urban system* a stable equilibrium configuration in which cities of different sizes co-exist or in which some regions develop cities whereas others do not. As we shall see, the analysis of such asymmetric equilibria is more involved because cities either inhibit or favor the existence of other cities in complex ways (Fujita *et al.*, 1999a, 1999b; Tabuchi *et al.*, 2005). We assume in the main text, as in the previous section, that regions are ex-ante symmetric, i.e., the fundamentals of all regions are the same. This allows us to derive some clear results. However, since our framework allows us to derive sharp general (equilibrium) conditions and to take them to the data, we will work with less restrictive assumptions and relax symmetry when doing so. This is another desirable and, to the best of our knowledge, original contribution of our model.

5.1 'Cannibalisation' effects

We first analyse in more detail the so-called short-run equilibrium condition (11).[29] The motivation builds on our postulate that this condition holds virtually at all times because prices and quantities adjust swiftly. For any number of regions/cities, this condition reads (for all

[29]We impose the long-run equilibrium condition $\mathbb{E}(\Delta V_l) \leq 0$, where $\mathbb{E}(\Delta V_l)$ is from (12), from Subsection 5.3 onwards.

$l = 1, 2, ..., \Lambda$):

$$\frac{1}{A\eta}\frac{\alpha - c_l}{c_l^{1+k}} = H_l + \phi \sum_{h \neq l} H_h \tag{25}$$

where the right hand side of the expression above is a measure of the so-called 'market potential' of city l (Head and Mayer, 2004b). Rewriting the system in matrix form yields

$$\underbrace{\begin{bmatrix} 1 & \phi & ... & \phi \\ \phi & 1 & ... & \phi \\ ... & & & ... \\ \phi & ... & \phi & 1 \end{bmatrix}}_{\mathbf{F}} \underbrace{\begin{bmatrix} H_1 \\ H_2 \\ ... \\ H_\Lambda \end{bmatrix}}_{\mathbf{h}} = \underbrace{(A\eta)^{-1} \begin{bmatrix} (\alpha - c_1)c_1^{-(1+k)} \\ (\alpha - c_2)c_2^{-(1+k)} \\ ... \\ (\alpha - c_\Lambda)c_\Lambda^{-(1+k)} \end{bmatrix}}_{\mathbf{x}} \tag{26}$$

where $\mathbf{F}$ is a Λ-dimensional invertible square matrix whose determinant $\det(\mathbf{F})$ is positive by inspection (all its off-diagonal elements are smaller than its diagonal elements) and $\mathbf{h}$ and $\mathbf{x}$ are both Λ-dimensional vectors. We can use (26) to prove the following proposition:

Proposition 8 (size and selection in an urban system) *Assume that regions are ex ante (or fundamentally) symmetric, i.e., they face the same bilateral trade costs and have identical ability supports. Then, at any equilibrium, selection is tougher in larger cities:*

$$c_l < c_h \iff H_l > H_h, \quad l \neq h.$$

Furthermore, $\partial H_l/\partial c_l < 0$ *and* $\partial H_l/\partial c_h > 0$.

Proof. See Appendix D.1. ■

In words, Proposition 8 establishes two important results. First, selection is tougher in larger agglomerations, i.e., the qualitative relationship established in (14) as a comparative statics exercise carries over to *an equilibrium relationship in an urban system.* As a corollary of this result, the positive relationship between the number of available varieties and the toughness of selection in (6) implies a hierarchy of cities akin to the *Central Place Theory* of Lösch (1940). Second, own city size decreases with the own cutoff (selection) and increases with the foreign cutoffs (competition). Insofar as a large H_l is the flip side of a low c_l, this finding suggests that urbanization in l may hinder urbanization in h and vice versa; we refer to this as the *'cannibalisation' effect* of proximate city productivity (see Dobkins and Ioannides, 2000).

To check first whether the correspondence between city productivity $1/c_l$ (where we measure productivity by the median household income in the MSA) and city size is borne out in the data, we first look at the (unconditional) Spearman rank correlation coefficient for our sample of U.S. CBSAs. Computing this coefficient for the 507 observations yields .4616, which is statistically positive at any conventional significance level.[30] Hence, larger cities are more productive. Obviously, the correlation is imperfect, as should be expected since the assumptions of equal trade

[30] The test uses the t-approximation for the Spearman rank correlation (see Zar, 1972).

costs and equal productivity supports are unlikely to be satisfied in reality. Since the model predicts that $(H_l - H_h)(c_l - c_h) \leq 0$ for all city pairs $h \neq l$, we may compute as an alternative test the percentage of correct predictions in the data. Out of the $\Lambda(\Lambda-1)/2 = 128,271$ possible pairs, we have $84,495$ positive signs, i.e., about 66% of correct predictions. This statistically exceeds the 'coin-toss outcome' at any conventional significance level.[31] It suggests again that there are systematic patterns in the data that are consistent with the short run equilibrium of the (ex ante) symmetric model, but also that differences in bilateral trade costs and/or productivity supports play a role in explaining the residual variation observed in the data.

Next, the theoretical 'cannibalisation' effect of proximate city size can be unveiled by explicitly accounting for spatial interdependence in the whole system of cities. To this end, we explicitly allow for different trade costs across pairs of cities. As shown in Appendix A.5, we may rewrite the equilibrium condition (25) for the general asymmetric case for empirical purposes, as follows:

$$H_l = \frac{1}{A\eta} \frac{\alpha - c_l}{c_l^{1+k}} - \sum_{h \neq l} \phi_{hl} H_h$$

where $\phi_{hl} \equiv \tau_{hl}^{-k}$. Let us linearise this expression around $c_l = \alpha$ to get

$$H_l \simeq \frac{1}{A\eta} \frac{\alpha - c_l}{\alpha^k} - \sum_{h \neq l} \phi_{hl} H_h.$$

The foregoing expression defines a structural, theory-based autoregressive spatial system (henceforth SAR; see Lee, 2004) of the following form:

$$\mathbf{h} = \mathbf{X}\beta + \rho\,\mathbf{W}\mathbf{h} + \varepsilon, \tag{27}$$

where $\mathbf{W}$ is the weight matrix (the weights are the trade frictions ϕ_{hl}); where $\mathbf{X}$ is a matrix of non-lagged variables (the cutoffs c_l plus a constant term); and where city size $\mathbf{h}$ is the endogenous spatially lagged variable.

We estimate β and the spatial autoregressive coefficient ρ using both standard quasi-maximum likelihood (QML) methods and Bayesian techniques.[32] To do so, we firstly restrict ourselves to

[31] Define the random variable $\mathcal{S} = \sum_i \sum_{j<i} \text{sgn}\left[\max\left\{0, (H_i - H_j)(c_i - c_j)\right\}\right]$, which is binomially distributed with $\Lambda(\Lambda-1)/2$ draws and probability 1/2. The null hypothesis for the coin-toss outcome is then given by $H_0 : \mathcal{S} = \Lambda(\Lambda-1)/4$, whereas the alternative hypothesis is given by $H_1 : \mathcal{S} > \Lambda(\Lambda-1)/4$. The number of pairs $\Lambda(\Lambda-1)/2$ is large enough so that we may use the normal approximation to the binomial distribution. Hence, $\mathcal{S}$ is significantly greater than $\Lambda(\Lambda-1)/4$ at the 1 percent (resp., 5 percent) level when $z \equiv \left[\mathcal{S} - M(M-1)/4\right]/\sqrt{M(M-1)/8}$ exceeds 2.33 (resp., 1.65).

[32] Note that, in the notation of (26), (27) is the (stochastic) first-order Taylor expansion of $\mathbf{F}\mathbf{h} = \mathbf{X}\beta$, with $\mathbf{F} = \mathbf{I}_\Lambda - \rho\mathbf{W}$ and $\mathbf{X} = [\mathbf{i}_\Lambda\ \mathbf{x}]$, where $\mathbf{I}_\Lambda$ is the $\Lambda \times \Lambda$ identity matrix and $\mathbf{i}_\Lambda$ is the Λ-row identity vector. Note also that the spatial autoregressive coefficient ρ need not lie in the range $(-1, 1)$ since $\mathbf{W}$ is not row-standardized. However, we use a numerical ajustement to make sure that the matrix $\mathbf{F}$ is invertible and the QML estimates are accurate.

the 367 Metropolitan Statistical Areas only and exclude the Micropolitan areas from our sample. Indeed, there are issues of heteroscedasticty in the sample, and including the Micropolitan areas exacerbates these problems in the estimations (sizes vary by a factor of almost 300 in the full sample). Secondly, we provide estimates for the full sample including the Micropolitan areas as a robustness check. In what follows, we use median income as a measure of productivity, itself an inverse measure of the cutoffs. We hence expect a positive β-coefficient on this variable. We also expect that ρ be negative, i.e., cities inhibit each others' growth when they are close enough ('agglomeration shadow'). We measure the trade frictions τ_{ij} by the great-circle distance between CBSAs. Constructing ϕ_{ij} then requires applying a negative power transformation, with exponent $k \geq 1$. As we have no a priori information on k, but since we work at an extremely aggregated level, we expect that k is close to 1 (see Axtell, 2001, for the U.S. size distributions of firms). Finally, we expect that ρ becomes mechanically more negative as k increases (since this reduces the ϕ_{ij}'s by exacerbating competition).

Insert Table 2 about here.

Table 2 presents estimation results for the SAR specification (27), using both QML (given in columns (i)—(v)) and Bayesian techniques (given in columns (vi)—(ix)). Note, first, that as expected, the coefficient on median income has a positive sign and is highly significant in all specifications. More productive cities are bigger. Note also that the spatial autoregressive coefficient is strictly negative in all specifications. However, it is only significant at the 10% level using QML estimates for the 367 MSAs, and it becomes insignificant when using the full sample. Thus, the 'cannibalisation' effect shows up in the data, at least when using the MSA subsample: being close to large (and thus productive cities) has a negative impact on own size. A by-product of this finding is that OLS estimates of the relationship between size and productivity are biased. Indeed, running the simple OLS regression without spatial interdependence yields a coefficient for median income of about .0553, which is smaller than the true coefficient because of ommited variable bias due to cannibalisation. Finally, we control in specification (v) for the potential influence of past dynamics on present city size. As can be seen, population growth in 2005-2006 has a negative impact on size in 2006, thereby suggesting that smaller cities have grown faster. Yet, the main qualitative findings are again unaffected.

When controlling for outliers and heteroscedasticity, using Bayesian estimation, the main qualitative findings of the model still hold true, though the magnitudes of the spatial autoregressive coefficient and of median income fall. However, all estimates are much more precise, even when using the full sample of 507 observations. This suggests that the relationship is significantly influenced by the largest and smallest MSAs, and that the relationship for those 'extreme' observations may deviate from that in the rest of the sample (as in Zipf's law of city size distributions; see e.g. Ioannides and Overman, 2003 and Duranton, 2007).

5.2 Market potential, accessibility, and polarisation

How do city-level income inequalities behave in a system of cities? In Section 4, we saw that the expression for the Gini coefficient in the symmetric case was already too unwieldy to be worth reporting in the main text. Matters obviously get even more obscure in a system of asymmetric cities. However, the real world is deeply and fundamentally asymmetric.[33] Therefore, we again use U.S. CBSA data to establish the *empirical* nature of the relationship between urban income inequalities and a city's market potential or accessibility to other cities.

To begin with, we run simple 'market potential regressions' where own size and the distance weighted size to the other cities is taken into account and estimate the short-run equilibrium system. Formally, we define the market potential as $\mathrm{mp}_j = \mathrm{pop}_j + \sum_{i \neq j}(\mathrm{pop}_i/d_{ij})$, which is identical to the right-hand side of (25) with $k = 1$. Columns (i)–(iii) in Table 2 summarise our results. As can be seen, market potential has a *positive* impact on income inequality. In words, larger local market size augmented by distance-weighted foreign markets, increases inequality. This result is robust across both MSA and CBSA samples and to the inclusion of our standard controls. Note, however, that the coefficient on market potential is *smaller* than that on purely local size (compare, e.g., (i) in Table 1 with (i) in Table 2). Put differently, distance-weighted access to other cities serves to partly reduce income inequality.

We next construct the average distance of each city from all the other cities (a measure of accessibility) and include it as an explanatory variable in the Gini coefficient regressions of the foregoing section. Columns (iv)–(viii) in Table 3 summarize our results.

Insert Table 3 about here.

As can be seen from columns (iv) and (v), a larger average distance from all other cities (worse accessibility) actually raises the Gini coefficient, yet leaves the impacts of all other variables unchanged. To understand what drives this result, specifications (vi) and (vii) reestimate the model by excluding all CBSAs with an average distance of more than 3000 kilometers from the rest of the cities.[34] As one can see, excluding the very remote regions, for which inequality might be driven by structurally very different mechanisms than mainland U.S., leads to insignificant positive point estimates of average distance on the Gini coefficients. Accessibility seems, therefore, to play an insignificant role in shaping urban inequality.[35] It is worth stressing that this result may stem from the fact that Proposition 7, which is a comparative statics prediction,

[33] The fundamental importance of asymmetry has been recognised by the 2008 Nobel Prize in Physics, attributed to Makoto Kobayashi, Toshihide Maskawa and Yoichiro Nambu "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics" (Nobel Foundation, 2008).

[34] Doing so excludes 13 observations that are all located in Puerto Rico, Alaska or Hawaii.

[35] Alternatively, we have replaced the arithmetic average distance with the geometric and the harmonic average distances. The qualitative results are unchanged: average distance (however measured) has a positive and significant coefficient, which gets insignificant when excluding the 13 remote observations.

has been derived under the strong assumption of perfect symmetry. It is hence not clear how it needs to be modified to fit the asymmetries in the data in a cross-section of cities. Furthermore, the big polarised U.S. cities are predominantly located close to the coasts, thereby raising their average distance when compared to the smaller less polarised yet more accessible cities in the mid-west.

5.3 Core-periphery equilibria

In the remainder of this section, we study some properties of asymmetric long-run equilibria using conditions (11) and (12). Two natural questions arise in NEG models: are there equilibria in which one region has a city but the others have none? When is such an extreme form of *urban primacy* (know as a *core-periphery configuration* in the NEG) a stable equilibrium? To answer these questions, consider a setting with an arbitrary number $\Lambda \geq 2$ of regions and impose the conditions $H_l > 0$, $H_h = 0$ for all $h \neq l$, and $0 < c_l, c_h \leq \alpha$. First, by (25), $H_h = 0$ necessarily implies that

$$\frac{\alpha - c_l}{\alpha - c_h} \left(\frac{c_h}{c_l} \right)^{1+k} > \frac{1}{\phi}. \tag{28}$$

It is readily verified that $H_l > 0$ whenever the foregoing condition holds. Indeed, a sufficient condition for $H_l > 0$ is

$$\frac{\alpha - c_l}{\alpha - c_h} \left(\frac{c_h}{c_l} \right)^{1+k} > \phi$$

which is satisfied whenever (28) holds since $\phi < 1$. The foregoing argument only requires the short-run equilibrium relationship (11). To be part of a long-run equilibrium, the free-entry condition (12) must hold with equality for $H_l > 0$ (so that all entry opportunities have been arbitraged away in city l) and it must hold with a strict inequality for $H_h = 0$ (so that nobody has any incentive to 'start a city'). Formally, $\mathbb{E}(\Delta V_l) = 0$ and $\mathbb{E}(\Delta V_h) < 0$.

Consider first the simple case in which there are no net entry costs for becoming an entrepreneur ($f^E = 0$). Then we can show the following impossibility result:

Proposition 9 (no asymmetric equilibria in the simple case) *Assume that there are no net entry costs for becoming an entrepreneur ($f^E = 0$). Then there exists no equilibrium such that $H_l^* > 0$ and $H_h^* = 0$ for all $h \neq l$.*

Proof. See Appendix D.2. ■

Proposition 9 shows that no core-periphery equilibria exist in the simple case. More generally, it can be shown that there exist no other asymmetric equilibria with $H_l > 0$ and $H_h > 0$ (see Appendix D.2 for further details).

Consider next the more general case where $f^E > 0$. For simplicity, we first investigate the possible existence of a core-periphery configuration with two regions h and l. Clearly, $H_l^* > 0$,

$0 < c_l^*, c_h^* \leq \alpha$ and $H_h^* = 0$ constitute an equilibrum if the following set of conditions hold simultaneously for $\{H_l^*, c_l^*\}$ and $\{H_h^*, c_h^*\}$:

$$0 = \frac{A}{k+2} H_l c_l^{2+k} + \frac{\alpha - c_l}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_l\right] - \theta H_l - f^E \tag{29}$$

$$0 \geq \frac{\phi A}{k+2} H_l c_l^{2+k} + \frac{\alpha - c_h}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_h\right] - f^E, \quad \forall h \neq l \tag{30}$$

$$0 = -H_l + \frac{1}{A\eta}\frac{\alpha - c_l}{c_l^{1+k}} \tag{31}$$

$$0 = -\phi H_l + \frac{1}{A\eta}\frac{\alpha - c_h}{c_h^{1+k}}, \quad \forall h \neq l. \tag{32}$$

Furthermore, the condition for local stability associated with $H_l^* > 0$ (which is given by either $\partial(\mathbb{E}(\Delta V_l))/\partial H_l < 0$ at H_l^* or, equivalently, by $\partial(\mathbb{E}(\Delta V_l))/\partial c_l > 0$ at c_l^*) need to hold. Despite the absence of closed form solutions to the system (29)–(32), we can prove several results analytically. First, H_l^* and c_l^* are uniquely determined by (29) and (31) and these are invariant in ϕ. Second, the two short-run equilibrium conditions (31) and (32) imply that $c_l^* < c_h^*$ by virtue of $0 < \phi < 1$. Third, assume that f^E and θ are low enough so that the conditions for existence and stability of an urban equilibrium in the one-region case of Section 3 are fulfilled. Then, we can establish the following results.

Proposition 10 (core-periphery equilibria) *Assume that f^E and θ are low enough so that a stable urban equilibrium exists when $\phi = 0$. Then there exists a unique threshold ϕ^{sust} in $(0, 1)$, called the 'sustain point', such that a core-periphery equilibrium in the two-region case, with $H_l^* > 0$ and $H_h^* = 0$ for all $h \neq l$, exists if (and only if) $\phi < \phi^{sust}$ (i.e. inter-city trade costs are large enough).*

Proof. See Appendix D.3. ■

Proposition 10 establishes that, in the limit when both markets are fully integrated (costless inter-city trade), location is irrelevant for both profits and consumer surplus ($\Pi_l = \Pi_h$ and $\mathrm{CS}_l = \mathrm{CS}_h$) but remains relevant for the urban cost of living. This is because urban dwellers incur commuting and housing costs and these depend on H_l and H_h. Thus, city sizes may not differ by much when $\phi \simeq 1$ (i.e., $\tau \simeq 1$). More generally, this result shows that at the limit $\phi \to 1$ all cities must be symmetric at equilibrium; if they are not, some agents can relocate and increase their well-being, which is inconsistent with the definition of an equilibrium. In the opposite configuration (prohibitive inter-city trade costs), cities are in isolation and the conditions for existence of a core-periphery equilibrium boil down to the conditions for urbanisation of Section 3.

Note that this extreme form of urban primacy arises among ex-ante, or structurally, identical regions. The result with symmetric trade costs established in Proposition 10 is in line with the findings of Ades and Glaeser (1995) and Krugman and Livas Elizondo (1996), whereby urban primacy and trade openness (both internal and international) are negatively associated.

Last, one may wonder whether core-periphery equilibria exist when there are more than two regions. This issue is hard to tackle analytically, especially since stability of equilibria becomes more difficult to establish. However, we can readily construct numerical examples exhibiting stable extreme urban primacy, as shown in Appendix D.4.

5.4 Darwinian systems of cities

In this subsection we construct another asymmetric long-run equilibrium using analytical means. Specifically, we illustrate the existence of an agglomeration shadow of large urban centres by considering what we could call an *extreme Darwinian system of cities*. The existence of an agglomeration shadow is due to the mechanism uncovered in Proposition 8 of Section 5.1, for which we have found some empirical evidence in the foregoing.

Assume that $\Lambda = 4$ regions are evenly located around a circle (a 'racetrack' economy as in Fujita *et al.*, 1999b). Shipping goods takes place on the circumference of the circle only. Therefore, $\tau_{lh} = \tau^{\min\{|l-h|,|4+l-h|\}}$. Let us construct an equilibrium in which regions 1 and 3 only develop cities of equal size $H_l^* > 0$ that hinder the development the urbanisation of regions 2 and 4 ($H_2^* = H_4^* = 0$). The equilibrium conditions in this case are ($l \in \{1, 3\}$, $h \in \{2, 4\}$):

$$0 = \frac{(1+\phi^2)A}{2+k} H_l c_l^{k+2} + \frac{\alpha - c_l}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_l\right] - \theta H_l - f^E \tag{33}$$

$$0 \geq \frac{2\phi A}{2+k} H_l c_l^{k+2} + \frac{\alpha - c_h}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_h\right] - f^E, \quad \forall h \neq l \tag{34}$$

$$0 = -(1+\phi^2)H_l + \frac{1}{A\eta}\frac{\alpha - c_l}{c_l^{1+k}} \tag{35}$$

$$0 = -2\phi H_l + \frac{1}{A\eta}\frac{\alpha - c_h}{c_h^{1+k}}, \quad \forall h \neq l. \tag{36}$$

Comparing the system of equations (33)–(36) with (29)–(32), it is easy to see that the qualitative properties of the former are similar to those of the latter. Specifically, there exists a threshold ϕ^{Darwin} in $(0, 1)$, similar in nature to the sustain point above, such that the extreme Darwinian system of cities (with every other region developing a city) exists if and only if $\phi < \phi^{Darwin}$ (i.e. inter-city trade costs are large enough).

However, there is still a key difference. First, the equilibrium cutoff c_l^* is no longer invariant in ϕ: as trade freeness rises, firms in either Darwinian city face increased competition from firms in the other cities, and the least productive of them exit, hence $\partial c_l^*/\partial\phi < 0$. The effect of an increase of ϕ on the size H_l^* of the Darwinian cities is ambiguous: one the one hand, competition is tougher and failures rates are higher, which discourages entry, but on the other hand the market for goods expands with access to the other Darwinian city, which encourages entry.

To get some numerical idea of the Darwinian case, we can readily construct an example using the following parameter values for $\Lambda = 4$ regions: $\alpha = 17.2$, $\gamma = 2$, $\eta = 22.5$, $H_0 = 4$, $\tau = 1.5$, $k = 1.2$, $c_{\max} = 30$, $f^E = 12$, $\theta = 0.14$. The resulting equilibrium is such that $H_1^* = H_3^* = 0.8014$, $c_1^* = c_3^* = 9.8661$, and $c_2^* = c_4^* = 10.1837$. It is worth noting that, when compared to the numerical core-periphery example given in Appendix D.4, the sum of the sizes of the two cities is smaller than that in the case of extreme urban primacy. This is another manifestation of the agglomeration shadow, which puts downward pressure on city sizes.

6 Conclusions

Cities are places of phenomenal wealth creation: in any cross-section of cities, the elasticity of worker and firm productivity with respect to city size is positive and typically in the $3\% - 8\%$ range. Empirically, this is so because the most efficient firms and the most skilled workers self-select into large cities, which then make them even more productive via agglomeration economies. Cities are also polarised: income inequalities are typically increasing in the size of urban agglomerations, at least in the U.S. This paper has provided a model of *vertical differentiation of cities* to account for such facts simultaneously. It has then used our theoretical framework to shed light on phenomena such as urbanisation, city resilience and the cannibalisation effect of agglomeration shadows, whereby the proximity of a large metropolitan area inhibits the development of smaller cities. A key finding of the paper is that *large urban areas are more productive and more polarised* than smaller ones, and that the latter is a consequence of the selection effect that is in part responsible for the former.

We have also unveiled a set of stylised facts from the data that are consistent with the most distinctive equilibrium conditions of our model. As a result, we conjecture that our model may be well-suited as a basis for further studies related to issues on urbanisation, agglomeration, and polarisation. On the empirical front, as we have said, we view the evidence provided in the paper as suggestive only. In future work, we need to think more deeply about better identification strategies. We also plan to use micro data to understand whether large cities are more unequal because they increase the dispersion of incomes and wages for a given level of observed skills or types (the so-called 'residual' wage inquality), or whether they are more unequal as the result of the observable 'divergence of human capital levels across cities' (a composition effect).[36] Using individual data, and incorporating sorting according to skills and types into our theoretical framework, should also help us identify the effects that are predicted by our theory.

[36] See Berry and Glaeser (2005). On the 'residual' wage inequality, see e.g. Lemieux (2006) or Helpman *et al.* (2008) and the references therein for a discussion.

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Appendix A: Guide to calculations

A.1. Micro-foundations for urban costs

Assume that all city dwellers consume one unit of land, as in Alonso (1964). Assume further that the central business district (CBD) is located at $x = 0$, so that a city of size H_l stretches out from $-H_l/2$ to $H_l/2$. Without loss of generality, we normalize the opportunity cost of land at the urban fringe: $R_l(H_l/2) = R_l(-H_l/2) = 0$. Each city dweller commutes to the CBD at constant unit-distance cost $\xi > 0$. Hence, an agent located at x incurrs a commuting cost of $\xi|x|$. Because expected profits and consumer surplus do not depend on city location (see Section 2), the sum of commuting costs and land rent must be identical across locations at a residential equilibrium. This implies that

$$\underbrace{R_l\left(\frac{H_l}{2}\right)}_{=0} + \xi\frac{H_l}{2} = R_l(x) + \xi|x|,$$

which yields the equilibrium land rent schedule $R_l(x) = \xi\left[H_l/2 - |x|\right]$. The aggregate land rent is thus given by

$$\text{ALR}_l = \int_{-\frac{H_l}{2}}^{\frac{H_l}{2}} R_l(x) = \frac{\theta}{4}H_l^2.$$

When ALR_l is equally redistributed to all agents, equilibrium total urban costs are

$$\frac{\text{ALR}_l}{H_l} - R_l(x) - \xi|x| = -\frac{\xi}{4}H_l.$$

Letting $\theta \equiv \xi/4 > 0$ then yields our reduced form for urban costs.

A.2. Derivation of consumer surplus

Denote by $D_l \equiv \int_{\mathcal{V}_l} d_l(\nu)\mathrm{d}\nu$ the demand for all varieties of the differentiated good. The inverse demand of an agent of type $i = E$ for each variety ν of that good is obtained by maximizing (1) under (2) and can be expressed as follows:

$$p_l(\nu) = \alpha - \gamma d_l(\nu) - \eta D_l \tag{A.1}$$

whenever $d_l(\nu) \geq 0$. Denote by $\mathcal{V}_l^+ \subseteq \mathcal{V}_l$ the subset of varieties *effectively consumed* in region l. Expression (A.1) can be inverted to yield a linear demand system as follows:

$$q_l(\nu) \equiv H_l d_l(\nu) = H_l\left[\frac{\alpha}{\eta N_l + \gamma} - \frac{p_l(\nu)}{\gamma} + \frac{\eta N_l}{\eta N_l + \gamma}\frac{\overline{p}_l}{\gamma}\right], \ \ \forall \nu \in \mathcal{V}_l^+, \tag{A.2}$$

where $\overline{p}_l \equiv (1/N_l)\int_{\mathcal{V}_l^+} p_l(\nu)\mathrm{d}\nu$ stands for their average price. By definition, $\mathcal{V}_l^+$ is the largest subset of $\mathcal{V}_l$ satisfying

$$p_l(\nu) \leq \frac{1}{\eta N_l + \gamma}\left(\gamma\alpha + \eta N_l \overline{p}_l\right) \equiv p_l^d. \tag{A.3}$$

For any given level of product differentiation γ, lower average prices $\overline{p}_l$ or a larger number of competing varieties N_l increase the price elasticity of demand and decrease the price bound p_l^d defined in (A.3). Stated differently, a lower $\overline{p}_l$ or a larger N_l generate a 'tougher' competitive environment, thereby reducing the maximum price at which entrepreneurs still face positive demand.

A.3. Nash price equilibrium and profit functions

Let

$$\pi_{hl}(c) = \left[p_{hl}(c) - \tau_{hl}c\right] q_{hl}(c)$$

denote operating profits, expressed as a function of the firm's marginal cost c. The firms sets prices in order to maximise these profits for each market separately. Then, the profit maximizing prices and output levels must satisfy (for $h \neq l$, with τ substituted for by 1 otherwise):

$$p_{hl}(c) = \frac{\alpha\gamma + \eta N_l \overline{p}_l}{2(\gamma + \eta N_l)} + \frac{\tau c}{2} \quad \text{and} \quad q_{hl}(c) = \frac{H_l}{\gamma}\left[p_{hl}(c) - \tau c\right]. \tag{A.4}$$

Integrating the prices in (A.4) over all available varieties, summing across regions and rearranging yields the average delivered price in market l as follows:

$$\overline{p}_l = \frac{\alpha\gamma + \eta N_l \overline{p}_l}{2(\gamma + \eta N_l)} + \frac{\overline{c}_l}{2} \quad \Rightarrow \quad \overline{p}_l = \frac{\alpha\gamma + (\gamma + \eta N_l)\overline{c}_l}{2\gamma + \eta N_l}, \tag{A.5}$$

where

$$\overline{c}_l \equiv \frac{\tau}{N_l}\sum_h\left[\int_{\mathcal{V}_{lh}^+} c\,\mathrm{d}G(c)\right]$$

stands for the average delivered cost of surviving firms selling to l. Plugging (A.5) into (A.4), some straightforward rearrangements show that the Nash equilibrium prices can then be expressed as follows:

$$p_{hl}(c) = \frac{c_l + \tau c}{2}, \quad \text{where} \quad c_l \equiv \frac{2\alpha\gamma + \eta N_l \overline{c}_l}{2\gamma + \eta N_l}$$

denotes the *domestic cost cutoff in region l*. Only entrepreneurs with c 'sufficiently smaller' than c_l are productive enough to sell in city l. This can be seen by expressing q_{hl} in (A.4) more compactly as follows:

$$q_{hl}(c) = H_l \frac{c_l - \tau c}{2\gamma}. \tag{A.6}$$

Clearly, selling in a 'foreign' market l when producing in h requires that $c \leq c_l/\tau$, whereas the analoguous condition for selling in the 'domestic' market is given by $c \leq c_l$ (recall that $\tau_{ll} = 1$). In what follows, we denote by c_{hl} the 'export' cutoff for firms producing in region h and selling to region l. This cutoff must satisfy the zero-profit cutoff condition $c_{hl} = \sup\{c \mid \pi_{hl}(c) > 0\}$. From expressions (4) and (A.6), this condition can be expressed as either $p_{hl}(c_{hl}) = \tau c_{hl}$ or $q_{hl}(c_{hl}) = 0$, which yields:

$$c_{hl} = \frac{c_l}{\tau}.$$

Equation (5) implies that $c_{hl} \leq c_l$ since $\tau \geq 1$. Put differently, trade barriers make it harder for exporters to break even relative to their local competitors because of higher market access costs. Since $p_l^d = p_{ll}(c_l) = c_l$, the zero-profit cutoff condition (A.3) can be expressed as follows:

$$\frac{1}{\gamma + \eta N_l}\left(\gamma\alpha + \eta N_l \overline{p}_l\right) = c_l, \quad \text{with} \quad \overline{p}_l = \frac{\alpha\gamma + (\gamma + \eta N_l)\overline{c}_l}{2\gamma + \eta N_l}.$$

We can thus solve for the mass of entrepreneurs selling in region l as follows:

$$N_l = \frac{2\gamma}{\eta} \frac{\alpha - c_l}{c_l - \overline{c}_l}. \tag{A.7}$$

Using the Pareto parametrization of Section 2.3, the average price and the average cost in region l are computed as follows:

$$\overline{p}_l = \frac{1 + 2k}{2 + 2k} c_l \quad \text{and} \quad \overline{c}_l = \frac{k}{1 + k} c_l,$$

i.e., they are given by a scaling of the domestic cutoff. Using this expression, as well as (A.7), we can then express the mass of sellers in l as follows:

$$N_l \equiv \sum_h H_h G(c_{hl}) = \frac{2\gamma(k+1)(\alpha - c_l)}{\eta c_l},$$

where the first equality comes from the definition of N_l.

The consumer surplus is finally derived by substituting the equilibrium prices into (3).

A.4. Expected profits

The expected profit in region l in the symmetric case under the Pareto parametrization is given as follows:

$$\begin{aligned}\mathbb{E}(\Pi_l) &= \frac{1}{H_l}\left[\frac{H_l}{4\gamma}\int_0^{c_l}(c_l-c)^2\, H_l \mathrm{d}G_l(c)+\frac{H_h}{4\gamma}\int_0^{\frac{c_h}{\tau}}(c_h-\tau c)^2\, H_l \mathrm{d}G_l(c)\right]\\ &= \frac{(c_l^{\max})^{-k}[H_l c_l^{2+k}+\tau^{-k}H_h c_h^{2+k}]}{2\gamma(1+k)(2+k)}.\end{aligned}$$

Using this expression, and noting that neither the consumer surplus nor the urban costs depend on the entrepreneur's ability, we readily obtain expression (12).

A.5. Asymmetric case

Note that expression (12) is derived under the simplifying assumption of symmetry, which we impose in the main body of the text. However, for applied purposes we will assume that trade costs may differ across regions. The general expressions of the equilibrium conditions are then as follows. First, the zero-expected utility gap is given by

$$\begin{aligned}\mathbb{E}(\Delta V_l) &= \frac{c_{l,\max}^{-k}[H_l c_l^{2+k}+\sum_{h\neq l}\tau_{lh}^{-k}H_h c_h^{2+k}]}{2\gamma(1+k)(2+k)}\\ &\quad+\frac{(\alpha-c_l)}{2\eta}\left[\alpha-\frac{1+k}{2+k}c_l\right]-f^E-\theta H_l. \qquad \text{(A.8)}\end{aligned}$$

Second, the identity for the masses of sellers (6) can be rewritten as:

$$\frac{2\gamma(k+1)(\alpha-c_l)}{\eta c_l}\equiv H_l\left(\frac{c_l}{c_{l,\max}}\right)^k+\sum_{h\neq l}\tau_{hl}^{-k}H_h\left(\frac{c_l}{c_{h,\max}}\right)^k \qquad \text{(A.9)}$$

Conditions (A.8) and (A.9) constitute a system of 2Λ equations in the 2Λ unknowns $\{H_l\}_{l=1}^{\Lambda}$ (city sizes) and $\{c_l\}_{l=1}^{\Lambda}$.

Appendix B: Proofs for Section 3

B.1. Proof of Proposition 1

Recall Proposition 1: The function $f(\cdot)$ has either one or three positive roots, of which at most two are in $[0,\alpha)$. Consequently, there exist at most two stable equilibria: the urban equilibrium and the rural equilibrium. If no stable urban equilibrium exists, then the rural equilibrium is unique. Furthermore, the equilibrium associated with the smallest value of c_l (the largest H) is always stable.

Proof. Rewriting $f(\cdot)$ in decreasing order of its powers in c_l, we obtain:

$$f(c_l) = K_1 c_l^2 - K_2 c_l \pm K_3 + K_4 c_l^{-k} - K_5 c_l^{-k-1}$$

where all coefficients K_i are strictly positive. Note that the constant K_3 (which is associated with c_l^0) may a priori be positive or negative, hence the $\pm$ sign in front of it. As one can see, in all cases there are at most three sign changes (from positive to negative or vice versa) between the coefficients of the consecutive powers. Let the number of positive roots be n and the number of sign changes be s. By Laguerre's (1883) generalization of *Descartes' rule of signs*, we know that $n \leq s$ (i.e., there are at most as many positive roots as sign changes) and $(s-n)$ is an even number if $n < s$. Hence, there are either 3 or 1 positive roots in our case, which are all positive since $\lim_{c_l \to 0} f(c_l) = -\infty$. By continuity, and the changes in the signs of the derivatives when there are multiple roots, it follows that there at most two stable equilibria. Indeed, applying Laguerre's generalisation of Descartes' rule to the first and second derivatives of $f(\cdot)$ reveals that $f'(\cdot)$ changes sign at most twice and that $f''(\cdot)$ changes signs at most once. The final part of the proposition results from the fact that $f(\cdot)$ increases from $-\infty$ at $c_l = 0$. Hence, $\partial f(\cdot)/\partial c_l$ must be strictly positive at the smallest root (whenever one exists). To see that the third root of $f(\cdot)$ is outside the relevant range $[0, \alpha]$ (a $c_l > \alpha$ implies a negative city size, which does not make any economic sense), it is sufficient to know that $f(\alpha) = -f^E$ and $\lim_{c_l \to +\infty} f(c_l) = \lim_{c_l \to +\infty} f'(c_l) = +\infty$. Thus, the largest root of $f(\cdot)$ is (strictly) larger than α if (and only if) the parameter f^E is (strictly) positive. ■

B.2. Proof of Proposition 2

Recall Proposition 2: All stable equilibrium city sizes H^* are non-increasing in θ and non-decreasing in A and α. Put differently, lower commuting costs (lower θ), a better productivity support (lower $c_{\max}$ and thus higher A), more product differentiation (lower γ and thus higher A), and stronger preference for the differentiated good (larger α) all weakly increase city size at any stable equilibrium.

Proof. It is readily verified that, for any given value of c_l, $f(\cdot)$ is strictly decreasing in $c_{\max}$ and θ. Assume that $c_l^* \in (0, \alpha]$ is a stable equilibrium. Two cases may arise: either $f(c_l^*) \leq 0$ (with $c_l^* = \alpha$), which corresponds to the rural equilibrium; or $f(c_l^*) = 0$, such that we are at an urban equilibrium. Consider an increase in θ (or in $c_{\max}$). Then since $f(\cdot)$ shifts down everywhere, it must be that $f(c_l^*) < 0$ in the first case: the rural equilibrium remains stable, and $H^* = 0$ is non-increasing from its initial value (which is also 0). In the second case, $f(c_l^*) < 0$ after the shift. Since stability implies that $\partial f(c_l^*)/\partial c_l > 0$, and since $f(\cdot)$ is continuous, the equilibrium must lie on the right of the previous one (hence a larger c_l^*, i.e., a smaller H^*). Of course, this encompasses the case where the new H^* can decrease to zero as we hit the corner and switch from an urban to a rural equilibrium.

Note finally that larger α and smaller γ both increase all stable equilibrium city sizes unambiguously. The proof is identical to the foregoing. ■

B.3. Complement to the proof of Proposition 4

To see that the condition $\theta > \theta^R$ is also *necessary* to ensure that there exists no pair $\{H, c_l\}$ with $c_l \in (0, \alpha)$ that is compatible with an equilibrium, we may proceed as follows.

Proof. First, we evaluate (15) at $\{\theta, f^E\} = \{\theta^U, 0\}$. The resulting free-entry condition, evaluated with equality, can be written as $2\alpha^{2+k}/(2+k) - \alpha c_l^{1+k} + c_l^{2+k}(k-1)/(k+2) = 0$. It follows from Laguerre's (1883) result that this expression has at most one root in $(0, \alpha)$. Invoking next its continuity, it is then straightforward to show that the LHS of this expression has exactly one root in $c_l \in (0, \alpha)$ by noting that it is positive when evaluated at $c_l = 0$ and negative when evaluated at $c_l = \alpha$. Second, we then evaluate (15) at $\{\theta, f^E\} = \{\theta^R, 0\}$. The resulting free-entry condition, evaluated with equality, can be written as $3\alpha^{2+k}/(2+k) - \alpha c_l^{1+k} + c_l^{2+k}(k-1)/(2+k) = 0$. Using the same arguments as in the foregoing, it is then straightforward to show that the LHS of this expression has a unique root at $c_l = \alpha$ by noting that the term in the square bracket is positive when evaluated at $c_l = 0$ and nil when evaluated at $c_l = \alpha$, and by continuity of this expression. Therefore, since (15) is continuous in both θ and c_l, it must be that, if $\theta < \theta^R$, then (15) admits a solution for c_l in $(0, \alpha)$, which establishes the necessary part. ■

B.4. Gini coefficients

In this appendix, we derive the Gini coefficient of income inequality (18). First, aggregate income in city h across all draws c is given by

$$W_h(c_h) \equiv H_h G(c_h) \int_0^{c_h} \frac{H_h}{4\gamma}(c_h - c)^2 \mathrm{d}G_h(c) = \frac{H_h^2 c_h^{2+k} c_{\max}^{-k}}{2\gamma(1+k)(2+k)}$$

since all agents with $c \geq c_h$ have zero income. The total income accruing to agents with draw $x \leq c_h$ is thus

$$W_h(x) \equiv H_h G(c_h) \int_0^{x} \frac{H_h}{4\gamma}(c_h - c)^2 \mathrm{d}G_h(c) = \frac{H_h^2}{4\gamma}\left(\frac{x}{c_{\max}}\right)^k \left(c_h^2 - \frac{2k}{1+k}x + \frac{k}{2+k}x^2\right),$$

and their income share is $W_h(x)/W_h(c_h)$. To compute the Gini coefficient, we have to link the income share with the population share. To do so, we need to switch to the distribution in terms of population shares (and not in terms of cost levels c). Let $y \equiv (x/c_{\max})^k$, i.e., $x = y^{1/k}c_{\max}$. Using this change in variables, the new upper bound for integration is given by $y = (c_h/c_{\max})^k$, and we obtain the integral of the Lorenz curve for the surviving agents as follows:

$$\int_0^{\left(\frac{c_h}{c_{\max}}\right)^k} \frac{W_h(y)}{W_h(c_h)}\mathrm{d}y - \int_0^{\left(\frac{c_h}{c_{\max}}\right)^k} x\mathrm{d}x = \frac{2+7k}{4+8k}\left(\frac{c_h}{c_{\max}}\right)^k - \frac{1}{2}\left(\frac{c_h}{c_{\max}}\right)^{2k} \qquad \text{(B.1)}$$

To finally obtain the Gini coefficient, we need to add the integral of the Lorenz curve for the agent who do not produce. This is given by

$$\int_{\left(\frac{c_h}{c_{\max}}\right)^k}^{1}(1-x)\mathrm{d}x = \frac{1}{2}\left[\left(\frac{c_h}{c_{\max}}\right)^k - 1\right]^2 \tag{B.2}$$

Summing (B.2) and (B.1) then yields the Gini index as follows:

$$\mathrm{Gini}_h(k;c_h) = 1 - \frac{2+k}{2+4k}\left(\frac{c_h}{c_{\max}}\right)^k. \tag{B.3}$$

Appendix C: Proofs for Section 4

C.1. Proofs of Propositions 11 to 14

In this Appendix we state and prove Propositions 11, 12, 13 and 14, which are respectively equivalent to Propositions 1, 2, 3 and 4 as given in Section 3.

Proposition 11 (number of symmetric equilibria) *Let $f^E \geq 0$. Then the function $f(\cdot)$ in (21) has at either one or three positive roots, of which at most two are in $[0,\alpha)$. Consequently, there exist at most two stable equilibria: the urban equilibrium and the rural equilibrium. If no stable urban equilibrium exists, then the rural equilibrium is unique. Furthermore, the equilibrium associated with the smallest value of c_l (the largest H_l) is always stable.*

Proof. Straightforward extension of the proof of Proposition 1. ■

Proposition 12 (monotonicity of urban equilibria) *If the interior equilibrium exists, then the (stable) equilibrium city size H^* is non-increasing in θ and non-decreasing in A and α. Put differently, lower commuting costs (lower θ), a better productivity support (lower $c_{\max}$ and thus higher A), more product differentiation (lower γ and thus higher A), and stronger preference for the differentiated good (larger α) each increase city size at the stable equilibrium.*

Proof. Straightforward extension of the proof of Proposition 2. ■

When does which type of equilibrium arise?

Proposition 13 (symmetric equilibria without cities) *The symmetric equilibrium without cities ($H_l^* = 0$ and $c_l^* = \alpha$) exists and is stable for all $f^E > 0$. When $f^E = 0$, it still exists but is a stable equilibrium iff $\theta \geq \theta^U$.*

Proof. This proposition is a simple corollary to Proposition 3. ■

The intuition of Proposition 3 carries over to Proposition 13. In addition, when there are no net opportunity costs for becoming an entrepreneur, the rural equilibrium exists and may be stable provided that trade costs are high (low Φ), suggesting that high trade and transportation costs are an impediment to the rise of trading cities. When there are no net entry costs for becoming an entrepreneur, (19) is equivalent to

$$\frac{2\theta}{A(1+\Phi)} \geq c_l^{1+k}\left(\alpha - \frac{k-1}{2+k}c_l\right), \tag{C.1}$$

the right-hand side of which is strictly concave in c_l, increasing in c_l at the limit $c_l \to 0$ and its maximum value is given by $3\alpha^{2+k}/(2+k)$. Hence the condition

$$\theta < \frac{3}{2}A(1+\Phi)\frac{\alpha^{2+k}}{2+k} = \frac{3}{2}\theta^U \equiv \theta^R$$

makes sure that there exists $c_l^* \in (0, \alpha)$ that is an equilibrium.

Proposition 14 (city resililence) *Assume that there are no net entry costs for becoming an entrepreneur ($f^E = 0$). Then: (i) $H^* = 0$ is the only stable equilibrium for all $\theta > \theta^R$; (ii) there exists a $\{H^*, c_l^*\} \in \mathbb{R}_{++} \times (0, \alpha)$ that is the unique stable equilibrium for all $\theta < \theta^U$; and (iii) for $\theta^U < \theta < \theta^R$, both $H^* = 0$ and some $\{H^*, c_l^*\} \in \mathbb{R}_{++} \times (0, \alpha)$ are stable equilibria.*

Proof. Straightforward application of the foregoing results. ■

C.2. Gini coefficients and trading cities

Let $z(\Lambda, \tau, k) \equiv -\lambda(\Lambda, \tau, k)/2$ so that (22) may be rewriten as

$$\text{Gini}_l(\Lambda, \tau, k; c_l) = 1 + 2z(\Lambda, \tau, k)\left(\frac{c_l}{c_{\max}}\right)^k,$$

with $z(\cdot) < 0$ for all Λ, τ and k. Fastidious calculations similar to those leading to (18) in appendix B.4. yield

$$\begin{aligned} z(\Lambda, \tau, k) \;=\; & -1 + \frac{\phi}{2(1+2k)}\frac{(\Lambda-1)\left[(\tau-1)^2(1+2k)(2+k)(1+k) + 2(\tau-1)(2+k)(1+3k) + 2 + 7k\right]}{2\tau^2 + (\Lambda-1)\left[(\tau-1)^2(2+k)(1+k) + 2(\tau-1)(2+k) + 2\right]} \\ & + \frac{1}{2(1+2k)}\frac{(2+7k)\tau^2}{2\tau^2 + (\Lambda-1)\left[(\tau-1)^2(2+k)(1+k) + 2(\tau-1)(2+k) + 2\right]} \end{aligned}$$

from which it follows that $-2z(1, \tau, k) = (2+k)/(2+4k)$ and that $-2z(\Lambda, 1, k) = (2+k)/(2+4k)$.

Recall now Proposition 7: Let income inequality be measured by the Gini coefficient at the symmetric equilibrium. Then income inequality is decreasing in trade/transportation costs, namely $\partial\text{Gini}_l/\partial\tau < 0$.

Proof. Differentiating $z(\Lambda, \tau, k)$ with respect to τ yields:

$$\begin{aligned}
\frac{\partial z(\Lambda,\tau,k)}{\partial \tau} &= -\frac{k(\Lambda-1)}{1+2k}\left\{\frac{\phi\left[\tau^2\kappa_2+\tau\kappa_1+\kappa_0\right]}{\left\{(\Lambda-1)\left[\tau^2(1+k)(2+k)-\tau(2+k)2k+(1+k)k\right]+2\tau^2\right\}^2}\right. \\
&\quad +\frac{\tau(2+7k)\left[(\tau-1)(2+k)+1\right]}{\left\{(\Lambda-1)\left[\tau^2(1+k)(2+k)-\tau(2+k)2k+(1+k)k\right]+2\tau^2\right\}^2} \\
&\quad \left. -\frac{\partial\phi}{\partial\tau}\frac{1}{k}\frac{(\tau-1)^2(1+k)(2+k)(1+2k)+2(\tau-1)(2+k)(3k+1)+(2+7k)}{(\Lambda-1)\left[\tau^2(1+k)(2+k)-\tau(2+k)2k+(1+k)k\right]+2\tau^2}\right\}
\end{aligned}$$

where $\kappa_2 \equiv (\Lambda-1)(1+k)(2+k)^2 - 4k(2+k)$, $\kappa_1 \equiv 3(\Lambda-1)(1+k)(2+k) - 2k(7+2k)$ and $\kappa_0 \equiv (\Lambda-1)(2+k) - 6k$ all have ambiguous signs; therefore, the term in the first line of the RHS above cannot be signed a-priori. By contrast, the terms on the second and third lines are positive by inspection. However, if $\phi\left[\tau^2\kappa_2+\tau\kappa_1+\kappa_0\right]$ is negative, then it is larger than $\tau^2\kappa_2+\tau\kappa_1+\kappa_0$ and, adding the terms of the first and second lines, implies

$$\begin{aligned}
&\phi\left[\tau^2\kappa_2+\tau\kappa_1+\kappa_0\right]+\tau(2+7k)\left[(\tau-1)(2+k)+1\right] \\
&\quad > \tau^2\kappa_2+\tau\kappa_1+\kappa_0+\tau(2+7k)\left[(\tau-1)(2+k)+1\right] \\
&\quad = (2+k)(\Lambda-1)\left[(1+k)(2+k)(\tau-1)^2+3(1+k)(\tau-1)+1\right] \\
&\qquad +(2+k)\left[(2+3k)(\tau-1)^2+3(1+k)(\tau-1)+1\right] > 0
\end{aligned}$$

which in turn implies that $\partial z(\Lambda,\tau,k)/\partial\tau < 0$ for all Λ, τ and k.

We have already established in Proposition 6 that selection gets tougher as trade gets freer ($\partial c_l/\partial\tau > 0$), therefore $\partial \text{Gini}_l/\partial\tau \equiv 2\left[c_l(\cdot)/c_{\max}\right]^k\left\{\partial z(\cdot)/\partial\tau + z(\cdot)c_l^{-1}\partial c_l(\cdot)/\partial\tau\right\} < 0$. ∎

For the sake of completeness, note that

$$\begin{aligned}
\frac{\partial z(\Lambda,\tau,k)}{\partial \Lambda} &= -\frac{1}{1+2k}\left\{\frac{-\tau^2\phi\left[(1+k)(2+k)(1+2k)(\tau-1)^2+2(2+k)(1+3k)(\tau-1)+2+7k\right]}{\left\{(\Lambda-1)\left[\tau^2(1+k)(2+k)-\tau(2+k)2k+(1+k)k\right]+2\tau^2\right\}^2}\right. \\
&\quad \left. +\frac{(2+7k)\tau^2\left[(1+k)(2+k)(\tau-1)^2+2(2+k)(\tau-1)+2\right]}{2\left\{(\Lambda-1)\left[\tau^2(1+k)(2+k)-\tau(2+k)2k+(1+k)k\right]+2\tau^2\right\}^2}\right\} \\
&< -\frac{k}{1+2k}\frac{\tau^2(\tau-1)(2+k)\left[3(1+k)(\tau-1)+2\right]}{2\left\{(\Lambda-1)\left[\tau^2(1+k)(2+k)-\tau(2+k)2k+(1+k)k\right]+2\tau^2\right\}^2} < 0.
\end{aligned}$$

Therefore, *given* c_l, granting access to more urban markets increases wages of the unperforming exporters relative to the wages of the most productive ones; this positive effect is strong enough to overcome the negative one on income inequalities that arises as a result of the wages of all successful entrepreneurs going up. However, since selection gets tougher as trade gets freer ($\partial c_l/\partial\tau > 0$), the two effects work in opposite directions. Our numerical simulations suggest that the latter indirect effect always dominates the former, direct effect. More precisely, the fact that a larger Λ increases the Gini coefficient *is entirely due to the increase in selection.* By contrast, the fact that a lower τ increases the Gini is due to the increase in selection *and* to the increase of the profits of most productive entrepreneurs (exporters in particular) relative to those of the least productive entrepreneurs (the purely domestic producers and those who fail to produce, in particular).

Appendix D: Proofs for Section 5

D.1. Proof of Proposition 8

Recall Proposition 8: Assume that regions are ex ante (or fundamentally) symmetric, i.e. they face the same bilateral trade costs and have identical ability supports. Then, at any equilibrium, selection is tougher in larger cities: $c_l < c_h \iff H_l > H_h, \quad l \neq h$. Furthermore, $\partial H_l / \partial c_l < 0$ and $\partial H_l / \partial c_h > 0$.

Proof. Straighforward rearrangement of (25) yields

$$\frac{\alpha - c_l}{\alpha - c_h} \left(\frac{c_h}{c_l} \right)^{1+k} = \frac{(1-\phi)H_l + \phi \sum_{i=1}^{\Lambda} H_i}{(1-\phi)H_h + \phi \sum_{i=1}^{\Lambda} H_i}$$

which implies

$$c_l < c_h \iff H_l > H_h.$$

To get the second result, recall that the solution to the linear system $\mathbf{Fh} = \mathbf{x}$ is given by $\mathbf{h} = \det(\mathbf{F})^{-1}\text{cof}(\mathbf{F})^T\mathbf{x}$, where $\text{cof}(\mathbf{F})$ stands for the matrix of cofactors associated with $\mathbf{F}$. As a result,

$$\frac{\partial H_l}{\partial c_l} = \frac{\det(\mathbf{F}_{l,l})}{\det(\mathbf{F})}$$

where $\det(\mathbf{F}_{l,l})$ is the minor of the $(\Lambda - 1) \times (\Lambda - 1)$ square matrix cut down from $\mathbf{F}$ by removing its l^{th} column and its l^{th} row. The matrix $\mathbf{F}_{l,l}$, like $\mathbf{F}$, has only 1's on its main diagonal and ϕ off its main diagonal. Thus, its determinant is also positive, i.e. $\det(\mathbf{F}_{l,l}) > 0$. By the same token,

$$\frac{\partial H_l}{\partial c_h} = \frac{\det(\mathbf{F}_{h,l})}{\det(\mathbf{F})}.$$

Now, from the *Gaussian elmination* algorithm, we know that $\det(\mathbf{F}_{h,l}) = -\det(\mathbf{F}_{l,l})$ for $l \neq h$ since $\mathbf{F}_{h,l}$ and $\mathbf{F}_{l,l}$ differ by a column permutation only. Hence $\det(\mathbf{F}_{h,l}) < 0$, which completes the proof. ■

D.2. Proof of Proposition 9

Recall Proposition 9: Assume that there are no net entry costs for becoming an entrepreneur ($f^E = 0$). Then there exists no equilibrium such that $H_l^* > 0$ and $H_h^* = 0$ for all $h \neq l$.

Proof. Let $f^E = 0$. The free-entry equilibrium condition for h (evaluated at the prospective equilibrium $H_h = 0$ and $H_l > 0$) is given by

$$\mathbb{E}(\Delta V_h) = \phi A \frac{[H_l c_l^{2+k} + \sum_{i \neq l,h} H_i c_i^{2+k}]}{2+k} + \frac{(\alpha - c_h)}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_h\right] \leq 0$$

which never holds since $c_h \leq \alpha$, $\phi \sum_{i \neq l,h} H_i c_i^{2+k} \geq 0$ and $H_l > 0$. Thus asymmetric equilibria in which one region does not develop a city but the other does do not exist in the simple case where $f^E = 0$. ■

More generally, it can be shown that there exist no other asymmetric equilibria with $H_l^*, H_h^* > 0$ and $H_l^* \neq H_h^*$. The reason is that the loci $\mathbb{E}(\Delta V_l) = 0$ and $\mathbb{E}(\Delta V_h) = 0$ intersect only once in the zone where $H_l > 0$ and $H_h > 0$.

D.3. Proof of Proposition 10

Recall Proposition 10: Assume that f^E and θ are low enough so that a stable urban equilibrium exists when $\phi = 0$. Then there exists a unique threshold ϕ^{sust} in $(0, 1)$, called the 'sustain point', such that a core-periphery equilibrium in the two-region case, with $H_l^* > 0$ and $H_h^* = 0$ for all $h \neq l$, exists if (and only if) $\phi < \phi^{sust}$ (i.e. inter-city trade costs are large enough).

Proof. All results hold by continuity of expressions (29)–(32) Step (i): from Proposition 1, we know that (29) admits at most one stable root in the interval $(0, \alpha)$. Let us denote this root by c_l^* and assume that f^E and θ are small enough so that a root with these properties exists. Note that c_l^* is invariant in ϕ. Step (ii): (31) and (32) together imply an implicit function of c_l and c_h:

$$\frac{\alpha - c_l}{\alpha - c_h}\left(\frac{c_h}{c_l}\right)^{1+k} = \frac{1}{\phi}.$$

Evaluating this expression at c_l^* yields a unique $c_h^* = c_h(\phi; c_l^*)$, which is decreasing and convex in ϕ. It is also readily verfied that $c_h(0; c_l^*) = \alpha$ and $c_h(1, c_l^*) = c_l^*$. Step (iii): define $f_h^*(\phi)$ as the RHS of the free-entry condition (30) evaluated at $c_h^* = c_h(\phi; c_l^*)$, using (31) to substitute for H_l^* and the definition of $\text{CS}(\cdot)$ in (7) to keep the expression synthetic:

$$f_h^*(\phi) \equiv \phi \frac{(\alpha - c_l^*)c_l^*}{\eta(2+k)} + \text{CS}(c_h^*) - f^E.$$

Then,

$$f_h^*(0) = \text{CS}(\alpha) - f^E = -f^E < 0$$

so that the core-periphery outcome is a stable equilibrium at the limit $\phi = 0$. Also,

$$f_h^*(1) = \frac{(\alpha - c_l^*)c_l^*}{\eta(2+k)} + \text{CS}(c_l^*) - f^E = f_l^* + \theta H_l^* = \theta H^* > 0$$

where f_l^* is defined as the right-hand side of the free-entry condition (29) evaluated at c_l^* and H_l^*. Clearly, $f_l^* = 0$ by definition. This intermediate result thus establishes that the core-periphery outcome is not an equilibrium at the limit $\phi = 1$. Step (iv): standard algebra reveals that $f_h^*(\cdot)$ is convex and increasing in ϕ on $(0, \alpha)$. Thus, by continuity, there exists a unique $\phi^{sust} \in (0, 1)$ such that

$$f_h^*(\phi^{sust}) = 0$$

and $f_h^*(\phi^{sust}) < 0$ if (and only if) $\phi < \phi^{sust}$. ■

D.4. Some core-periphery results with $\Lambda > 2$

In this appendix we present the analytical descriptions of other core-periphery equilibria in the Λ-region case, where $\Lambda > 2$. We also provide numerical examples illustrating these types of equilibria. Assume that there are $\Lambda = 4$ regions which are symmetrically located, i.e., the bilateral cost is τ between any two pair of regions. Formally, a core-periphery configuration whereby $H_l^* > 0$, $0 < c_l^*, c_h^* \leq \alpha$ and $H_h^* = 0$, for all $h \neq l$, will be part of an equilibrum if the following conditions hold simultaneously for some H_l^*, c_l^* and $\{c_h^*\}_{h \neq l}$ in the relevant ranges:

$$\begin{aligned}
0 &= A\frac{(H_l + H_0)c_l^{2+k} + \phi H_0 c_h^{2+k}}{2+k} + \frac{\alpha - c_l}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_l\right] - \theta(H_l + H_0) - f^E \\
0 &\geq A\frac{(1 + \Phi - \phi)H_0 c_h^{2+k} + \phi(H_l + H_0)c_l^{2+k}}{2+k} + \frac{\alpha - c_h}{2\eta}\left[\alpha - \frac{1+k}{2+k}c_h\right] - \theta H_0 - f^E, \quad \forall h \neq l \\
0 &\equiv -\phi H_l + \frac{\alpha - c_h}{c_h^{1+k} A\eta}, \quad \forall h \neq l \\
0 &\equiv -H_l + \frac{\alpha - c_l}{c_l^{1+k} A\eta}.
\end{aligned}$$

Note that since we focus on the symmetric case, the second and the third conditions are identical for all the peripheral regions.

One can readily construct an example with extreme urban primacy using the following parameter values for $\Lambda = 4$ regions: $\alpha = 17.2$, $\gamma = 2$, $\eta = 22.5$, $H_0 = 4$, $\tau = 1.5$, $k = 1.2$, $c_{\max} = 30$, $f^E = 12$, $\theta = 0.14$. One resulting stable equilibrium is such that $H_1^* = 1.6632$, $c_1^* = 8.7397$, and $c_i^* = 10.0806$ for $i = 2, 3, 4$.[37]

The foregoing equilibrium conditions reveal that, given τ and the other parameters of the model, *this extreme form of urban primacy is less likely to arise at equilibrium the larger is* Λ. However, if it does arise, *the size of the primate city will be larger the larger* ϕ. The first

[37]When there are more than two regions, there are an infinity of possible perturbations of any equilibrium. In that case, as shown in the technical Appendix TA.2, we need to check that the Jacobian associated with the equilibrium conditions is negative definite. In the numerical applications presented in this paper, we check the full stability condition on the Jacobian of f (see footnote 22 in Section 4.2 for further details). The Mathematica code of the numerical procedure is available from the authors upon request.

equilibrium condition shifts up with ϕ, so that larger values of ϕ decrease c_l^* and, therefore, increase H_l^*. In words, primate cities grow as ϕ rises. Yet, the second condition shifts also up with Φ (hence, with Λ), thereby making this configuration less likely to remain an equilibrium. The freeness of trade (an exogenous measure of trade openness) increases in the number of trading partners. Therefore, increasing Λ should have qualititatively similar effects to increasing ϕ.

Switching to $\Lambda = 10$ regions, and keeping the same parameter values as before, we have still extreme urban primacy with $H_1^* = 4.4092$, $c_1^* = 6.2967$ and $c_i^* = 7.4619$ for $i = 2, 3, \ldots, 10$. The core grows bigger, as stated in the foregoing. However, when $\Lambda = 11$, extreme urban primacy is no longer feasible and we switch to a different equilibrium configuration with multiple core regions.

Technical appendix

The following appendix provides technical details on stability and on how to implement the numerical procedures for constructing different types of equilibria.

TA.1. Additional stability results

Combining (23) and (24) we obtain an expression for $\mathrm{d}\mathbb{E}(\Delta V_l)/\mathrm{d}\ln H$ as follows:

$$\frac{\mathrm{d}\mathbb{E}(\Delta V_l)}{\mathrm{d}\ln H} = -\theta H - \frac{1-\phi}{1+\Phi}\frac{\alpha - c_l}{2\eta}\frac{c_l}{2+k}\frac{\alpha - 2c_l}{\alpha + k(\alpha - c_l)} + \frac{1}{\eta}\left[\frac{1-\phi}{1+\Phi}(\alpha - c_l)\right]^2 \frac{c_l}{\alpha + k(\alpha - c_l)}$$

From this expression, observe that $\mathrm{d}\mathbb{E}(\Delta V_l) < 0$ at the limit $\tau \to 1$ ($\phi \to 1$). The net benefit of adding one entrepreneur to city l is converging to the urban cost at the rate $1 - \phi$. In addition, note that the gross benefit of adding one entrepreneur to city l is converging to zero at the rate $(1-\phi)^2$, i.e. twice as fast. Note that the sign of the second term is ambiguous: it is non-negative if the root $c_l \in (0, \alpha/2]$ and non-positive if $c_l \in (\alpha/2, \alpha]$. Going back to (23), we observe that the this term is the net effect of the backward (demand) linkage and the congestion/selection effects. A large market size is good for (expected) profits but at the same time it is associated with a tougher environment. As in Krugman (1991), entrepreneurs' nominal wages might actually be negatively linked to market size; this happens when a stable equilibrium arises for some $c_l \in (0, \alpha/2)$. In this cases, city l is a *consumer city* (in the sense of the previous section) and this type of equilibrium arises because a lower consumer price index (a larger consumer surplus) more than compensates urban dwellers for the urban costs they incur by living in a large city. When the lower consumer price index falls short of the larger urban costs, city l is a *producer city* with $c_l \in (\alpha/2, \alpha)$ arising at equilibrium . In a sense, the model is more realistic and more general than Krugman's in the sense that living in large urban area is associated with more expensive dwellings and commuting (see also Murata and Thisse, 2005).

As we cannot obtain closed form solutions for the equilibrium H and c_l, to make further progress, we totally differentiate (12) and evaluate it at the symmetric equilibrium. We obtain:

$$
\begin{aligned}
\mathrm{d}\mathbb{E}(\Delta V_l) &= \frac{A}{2+k}\left\{\mathrm{d}\left[Hc_l^{2+k}\right]+\phi\sum_{h\neq l}\mathrm{d}\left[H_h c_h^{2+k}\right]\right\}+\mathrm{d}\left[\frac{\alpha-c_l}{2\eta}\left(\alpha-\frac{1+k}{2+k}c_l\right)\right]-\theta\mathrm{d}H \\
&= \frac{(1+\Phi)A}{2+k}\mathrm{d}\left[Hc_l^{2+k}\right]+\mathrm{d}\left[\frac{\alpha-c_l}{2\eta}\left(\alpha-\frac{1+k}{2+k}c_l\right)\right]-\theta\mathrm{d}H \qquad \text{(TA.1)}\\
&\quad +\frac{(1+\Phi)A}{2+k}\sum_{h\neq l}\left\{\mathrm{d}\left[H_h c_h^{2+k}\right]-\mathrm{d}\left[Hc_l^{2+k}\right]\right\}\leq-\frac{(1+\Phi)A}{2+k}\mathrm{d}\left[Hc_l^{2+k}\right].
\end{aligned}
$$

The step to get to the second line follows from simple rearrangement of terms and by definition of Φ. The term in the first line of this expression is non-positive by the stability of the symmetric equilibrium in the sense of Proposition 14, which explains the inequality sign to get from the third to the fourth line of this expression. Thus, the symmetric equilibrium is stable at the limit $\tau=\infty$; this limiting case corresponds to the case studied in the previous section, so this result comes at no surprise.

As can be seen from (TA.1), an alternative sufficient condition for the symmetric equilibrium to be stable is given by $\mathrm{d}\left[Hc_l^{2+k}\right]/\mathrm{d}c_l>0$. When does this hold? To answer this question, rearrange (20) as follows:

$$
Hc_l^{2+k}=\frac{(\alpha-c_l)c_l}{(1+\Phi)A\eta} \qquad \text{(TA.2)}
$$

which is increasing in c_l for $c_l\in(0,\alpha/2)$ and decreasing in c_l for $c_l\in(\alpha/2,\alpha)$. Therefore, the system is stable regardless of the value of ϕ if the equilibrium c_l is in $(0,\alpha/2)$. Generically, there exists a positive measure of parameter combinations such that this condition is satisfied at equilibrium.

To obtain additional results, let us briefly consider the simple case ($f^E=0$). In this case, pluging (TA.2) into (12) yields

$$
\mathbb{E}(\Delta V_l)=f(c_l)\equiv\frac{\alpha-c_l}{\eta}\left[\frac{c_l}{2+k}+\frac{1}{2}\left(\alpha-\frac{1+k}{2+k}c_l\right)-\theta c_{\max}^k\frac{2\gamma(1+k)}{1+\phi}c_l^{-(1+k)}\right]=0. \qquad \text{(TA.3)}
$$

By inspection, $c_l^*=\alpha$ is always a root of the equation $f(.)$. It follows from Laguerre's generalization of *Descartes' rule of signs* and by continuity that there is at most one root in $(0,\alpha)$ that corresponds to a stable equilibrium. Further analysis, available from the authors upon request, show that there exist a cutoff value for θ in $(0,\theta^R)$, denoted by $\theta_0^{\alpha/2}$, such that a stable equilibrium with $c_l^*\in(0,\alpha/2)$ exists if $\theta\in[0,\theta_0^{\alpha/2})$ and a stable equilibrium with $c_l^*\in[\alpha/2,\alpha]$ exists if $\theta\in[\theta_0^{\alpha/2},\theta^R]$.

TA.2. General procedure to check stability numerically

In the numerical application, we can check stability for any potential equilibrium candidate (including arbitrary corner solutions) more thoroughly as follows. Let Ω^+ and Ω^0 denote the sets of regions with and without a city at equilibrium, respectively. Assume that there are z regions without a city. The numerical procedure for constructing equilibria and for checking their stability is then as follows.

Let $\mathbf{c} = (c_1 \ c_2 \ \dots \ c_\Lambda)$ and let $\mathbf{H} = (H_1 \ H_2 \ \dots \ H_\Lambda)$. First, the non-positive expected profit is given by

$$\mathbb{E}(\Delta V_l) = \frac{A}{k+2} \sum_{h \in \Omega^+} \phi_{lh} H_h^* c_h^{*2+k} + \frac{(\alpha - c_l^*)}{2\eta} \left[\alpha - \frac{1+k}{2+k} c_l^* \right] - f^E - \theta H_l^* \equiv f_l(\mathbf{c}, \mathbf{H}) \leq 0 \quad \text{(TA.4)}$$

for any region. Condition (TA.4) must hold with equality for all $l \in \Omega^+$ and with strict inequality for all $l \in \Omega^0$. Second, for any l, the identity for the masses of sellers can be rewritten as:

$$g_l(\mathbf{c}, \mathbf{H}) \equiv A \frac{\alpha - c_l^*}{\eta c_l^{*1+k}} - \sum_{h \in \Omega^+} \phi_{hl} H_h^* \equiv 0 \quad \text{(TA.5)}$$

Conditions (TA.4) for $l \in \Omega^+$ and conditions (TA.5) for all $l = 1, 2, \dots, \Lambda$ constitute a system of $2\Lambda - z$ equations in the $2\Lambda - z$ unknowns $\{H_l\}_{\Omega^+}$ and $\{c_l\}_\Lambda$. Denote by $(\mathbf{c}^*, \mathbf{H}^*)$ a solution to that system. If $f_l(\mathbf{c}^*, \mathbf{H}^*) < 0$ for all $l \in \Omega^0$, this solution is an equilibrium candidate.

To check whether this solution is a stable equilibrium we proceed as follows. We can uniquely solve the set of equations $g_l(\mathbf{c}, \mathbf{H}) = 0$ for all $l \in \Omega^+$ for the $H_l = H_l(\mathbf{c}_{\Omega^+})$ for $l \in \Omega^+$. Note that $\mathbf{c}_{\Omega^+}$ denotes the $\Lambda - z$ dimensional vector of the $\{c_l\}_{\Omega^+}$. We can thus substitute out the $\{H_l\}_{\Omega^+}$ in (TA.4). Since $H_l = 0$ for $l \in \Omega^0$, we obtain a system of $\Lambda - z$ equations in the Λ variables c_l. To check whether no deviation from one city to another is profitable, we have to make sure that the Jacobian associated with $\{f_l\}_{\Omega^+}$ in the variables $\{c_l\}_{\Omega^+}$ is positive definite at $\mathbf{c}^*$ (recall that we already substituted out the H_l) on the subset generated by the constraints (TA.5) for $l \in \Omega^0$. After substituting the positive H_l into (TA.5), the constraints are given by

$$g_l(\mathbf{c}) \equiv A \frac{\alpha - c_l}{\eta c_l^{1+k}} - \sum_{h \in \Omega^+} \phi_{hl} H_h(\mathbf{c}_{\Omega^+}) \equiv 0 \quad \text{(TA.6)}$$

for all $l \in \Omega^0$. These constraints define on a one-to-one basis the equilibrium relationships between any c_i with $i \in \Omega^0$ and the set of variables c_l with $l \in \Omega^+$. Applying the implicit function theorem, we then obtain $\mathrm{d}c_l/\mathrm{d}c_i$ for $l \in \Omega^0$ and $i \in \Omega^+$. This finally allows to compute the $\Lambda - z$ square matrix of the Jacobian of $\{f_l\}_{\Omega^+}$ in $\{c_l\}_{\Omega^+}$, *taking into account the GE constraints via the* $\mathrm{d}c_l/\mathrm{d}c_i$ *terms*. It can readily be evaluated at the equilibrium candidate $(\mathbf{c}^*, \mathbf{H}^*)$. The equilibrium canditate is (locally) stable if this Jacobian is positive definite (which is the higher-dimensional extension of the simple stability condition $\mathrm{d}f(\cdot)/\mathrm{d}c_l > 0$ used in the simple cases).[38]

[38] The procedure can be readily implemented with Mathematica and the code is available from the authors upon request.

Table 1 — Size and income inequality

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
Year	2006	2006	2006	2006	2006	2006	2007
Obs.	367	507	367	367	367	358	510
Sample	MSA	CBSA	MSA	MSA	MSA	MSA	CBSA
size	.0106	.0136	.0091	.0108	.0090		.0136
	(.000)	(.000)	(.000)	(.000)	(.000)		(.000)
$size^2$	−.0004	−.0005	−.0003	−.0004	−.0003		−.0005
	(.007)	(.000)	(.027)	(.023)	(.028)		(.000)
medi	−.0015	−.0014	−.0014	−.0021	−.0020	−.0012	−.0015
	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)
black			.0519		.0601		
			(.000)		(.000)		
edu				.1394	.1529		
				(.000)	(.000)		
dens						.0383	
						(.000)	
Adj R^2	.2116	.1757	.2441	.2693	.3133	.1364	.1969

Notes: The dependent variable in all specifications is the household income Gini coefficient. p-values are given in parenthesis. All regressions include a constant term. Explanatory variables are defined as follows: size = population size of the MSA in millions; medi = household median income in inflation-adjusted 1000 US$; black = share of black or Afro-American population; edu = share of population with some college education or higher; dens = population in 1000 per square mile. All data is from the 2006 American Community Survey (U.S. Census Bureau), except for MSA surface data which is from the 2000 Census of Population and Housing (U.S. Census Bureau). Data for the 2007 American Community Survey are based on 1-year estimates.

Table 2 — 'Cannibalisation' effects and the agglomeration shadow

	Quasi-maximum likelihood					Robust bayesian estimates				
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Obs.	367	367	367	507	363	367	367	367	507	363
Sample	MSA	MSA	MSA	CBSA	MSA	MSA	MSA	MSA	CBSA	MSA
k	1	1.1	1.2	1	1	1	1.1	1.2	1	1
medi	.0574	.0576	.0576	.0498	.0573	.0356	.0356	.0359	.0276	.0356
	(.000)	(.000)	(0.000)	(.000)	(.000)	(.000)	(.000)	(0.000)	(.000)	(.000)
ρ	−1.3725	−2.1972	−3.4633	−.9285	−1.3321	−.8893	−.9051	−2.3714	−.5405	−.8632
	(.072)	(.068)	(.063)	(.391)	(.081)	(.029)	(.028)	(.031)	(.036)	(.036)
popg					−5.1201					−2.8287
					(.042)					(.022)
Pseudo R^2	.3390	.3401	.3411	.3334	.3519	.3348	.3349	0.3368	.3296	.3482

Notes: The dependent variable in all specifications is MSA size (in millions) in 2006. z-values in parenthesis. All regressions include a constant term (not shown) and use a numerical adjustment to control for the fact that the weight matrix is not row-standardized. Explanatory variables are defined as follows: medi = household median income in inflation-adjusted 1000 US$ for 2006; popg = population growth in % between 2005 and 2006. The distance matrix constructed as $\phi_{ij} = d_{ij}^{-k}$, where d_{ij} is great-circle distance in km between the metropolitan and/or micropolitan areas. Bayesian estimates are computed using 25000 MCMC draws, discarding the first 10000. The pseudo R^2 is computed as the linear correlation between y and $\widehat{y}$. All data is from the 2006 American Community Survey (U.S. Bureau of Census).

Table 3 — Market potential regressions

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
Obs.	367	367	499	507	499	494	356
Sample	MSA	MSA	CBSA	CBSA	CBSA	CBSA dist <3	MSA dist <3
size				.0134	.0093	.0124	.0091
				(.000)	(.000)	(.000)	(.000)
size^2				$-.0005$	$-.0003$	$-.0005$	$-.0003$
				(.000)	(.016)	(.000)	(.033)
mp	.0057	.0052	.0054				
	(.000)	(.000)	(.000)				
dist				.0061	.0091	.0018	.0011
				(.000)	(.000)	(.476)	(.701)
medi	$-.0014$	$-.0019$	$-.0019$	$-.0015$	$-.0020$	$-.0012$	$-.0012$
	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)
black		.0633	.0816		.0912		
		(.000)	(.000)		(.000)		
edu		.1551	.1983		.1885		
		(.000)	(.000)		(.000)		
Adj R^2	.1930	.3016	.3170	.1914	.3666	.1190	.1155

Notes: The dependent variable in all specifications is the household income Gini coefficient in 2006. p-values are given in parenthesis. All regressions include a constant term. Explanatory variables are defined as follows: size = population size of the MSA in millions; medi = household median income in inflation-adjusted 1000 US$; black = share of black or African American population; edu = share of population with some college education or higher; mp = market potential, defined as $\mathrm{mp_j} \equiv \mathrm{pop}_j + sum_{i \neq j}(\mathrm{pop}_i/d_{ij})$, and where d_{ij} is great circle distance in kilometers; dist = average distance to all other cities (thousand km). All data is from the 2006 American Community Survey (U.S. Census Bureau), except for MSA surface data which is from the 2000 Census of Population and Housing (U.S. Census Bureau).

Figure 1(a): Rural equilibrium is the unique equilibrium.

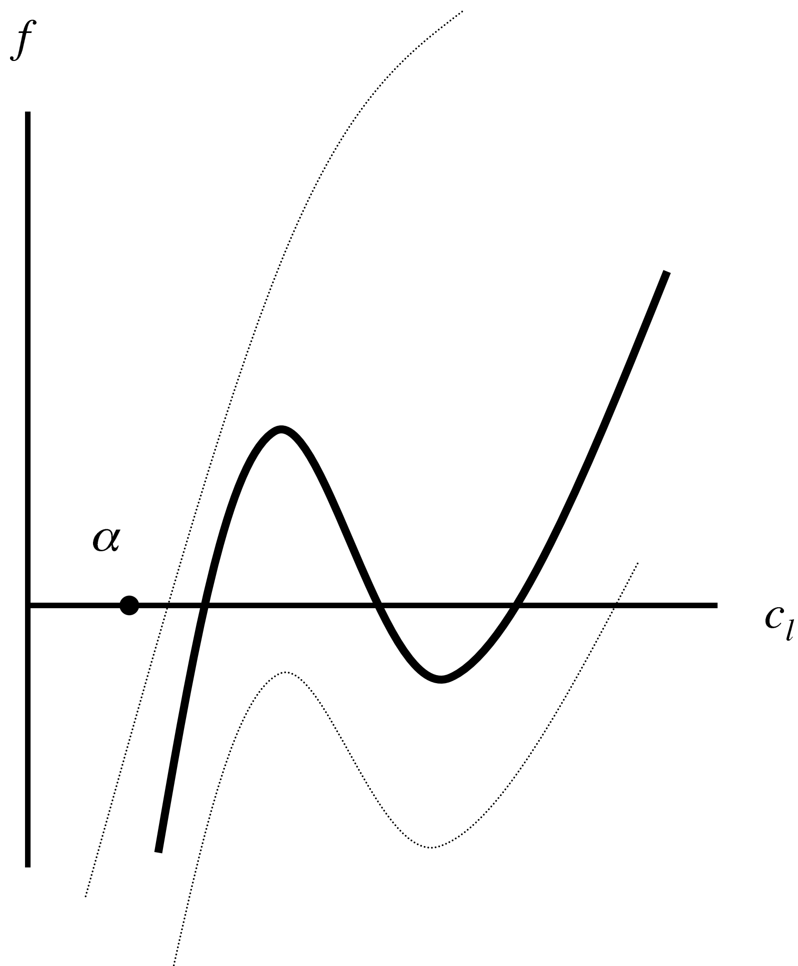

Figure 1(b): Urban equilibrium is the unique stable equilibrium

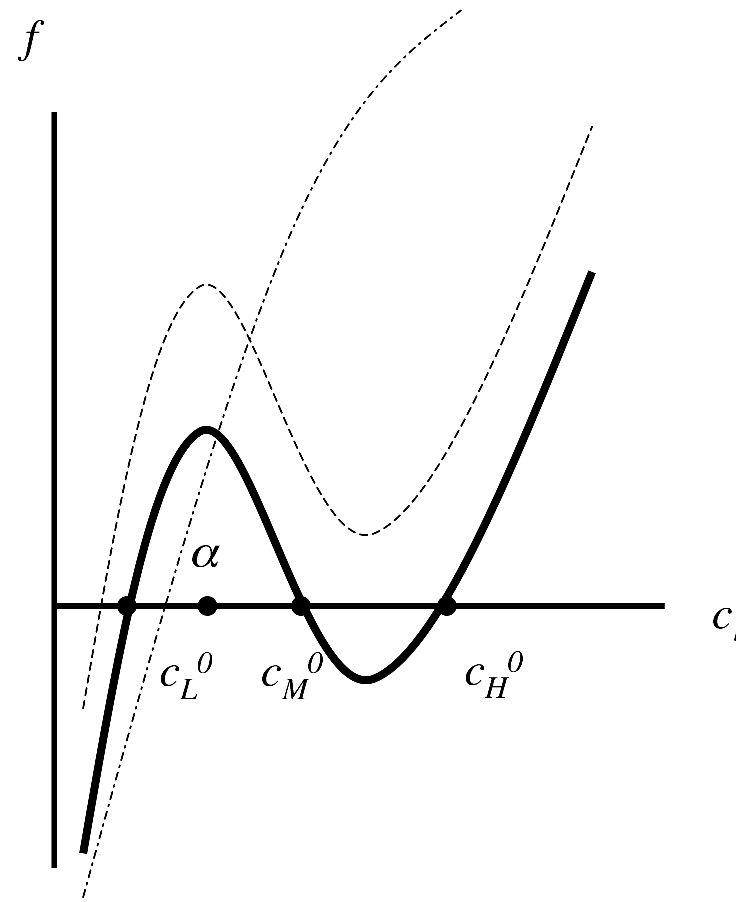

Figure 1(c): Two stable equilibria (rural and urban)

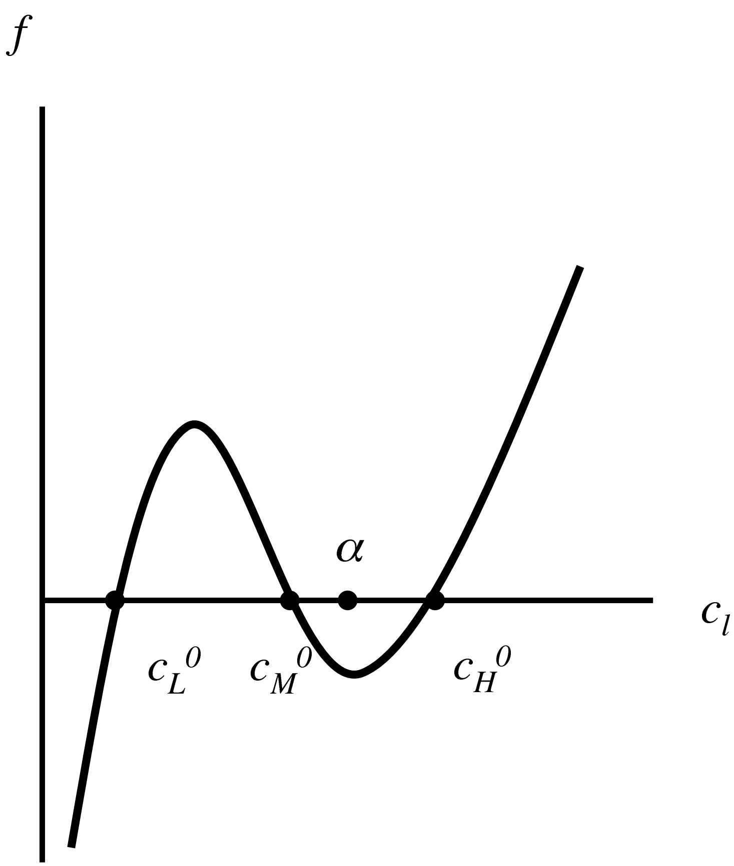

Figure 2: Equilibrium configurations with $f^E = 0$.

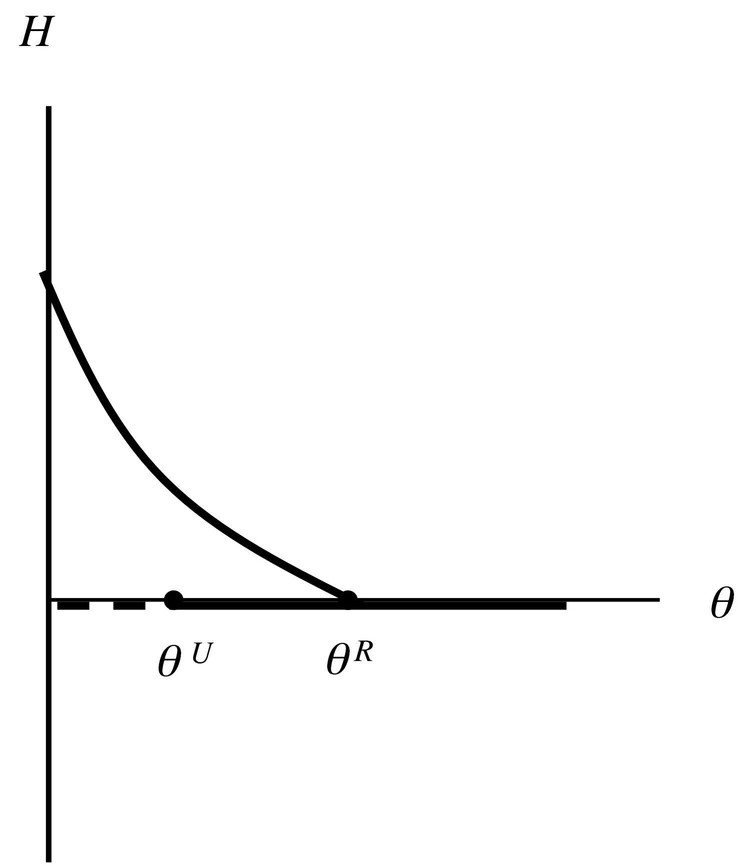

Figure 3(a): Gini coefficient as a function of trade costs using the stable c_l^* ($\Lambda = 4$)

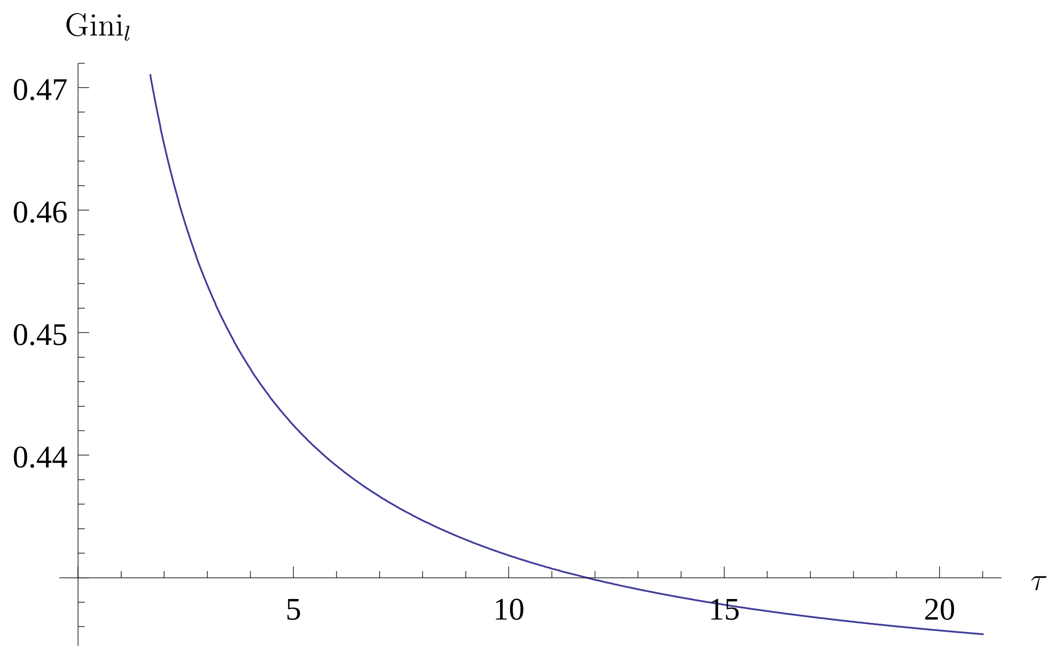

Figure 3(b): Gini coefficient as a function of trade costs using the stable c_l^* ($\Lambda = 10$)