Two versus many: the geographical dimension in NEG models

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Abstract
Results of core-periphery models dramatically change with the number of regions in which the national economy is divided and with its geographical structure. Concentration in an *n*-region setting can simply be reversed into dispersion by adding or deleting just one region. The influence of the geographical dimension is examined for the centre-periphery model (CP) and the footloose enterepreneur model (FE). The results show that the urban size distribution of the FE model can be modelled invariant to the number of regions, but only when the economic parameter space has a low agglomeration tendency. This is not true for the predicted spatial distribution: as *n* increases, the agglomerations are predicted on a different place. Compared with the FE model, the CP model produces both size and spatial distributions that are much less invariant to *n*.

1. Introduction

The geographical dimension of NEG models is essentially defined by the *nxn* distance matrix *D* between *n* regions, which together with the transport cost parameter *T* determines interregional transport costs[1]. In a two-region setting, however, there is only one geographical connection to travel for firms, workers and goods. Under the usual assumption that $d_{ij} = d_{ji}$ for any pair of regions *i* and *j D* then reduces to a scalar *d* that can be normalized to unity by adjusting *T*. For the two-region model this implies that its geography is trivial: the two regions can be anywhere at any distance. Adding a third region requires the inevitable choice to put it somewhere relative to the other two and is in fact the decision to let geography enter the stage or not. However, a minimum of three regions is a necessary but still not sufficient condition for a non-trivial geography. When all regions are at equal distance on a circle or a torus, no region has any locational advantage over the other. This option for a "neutral geography" Brakman, Garretsen, and Marrewijk (2008) allows a pure economic analysis without results that are biased by "non-neutral" geography.

From the geographers point of view the very concept of a neutral geography is a contradiction in terms. In three-dimensional reality locations can only be spatially neutral on a homogeneous torus or globe surface without oceans, rivers, mountains or any other obstacle for transport that would make some locations better accessible than others. In other words, for NEG models that take their "G" seriously, non-neutrality is the sine qua non of the geographical dimension, invariant to whether the original *core-periphery* (CP) model or other economic variants like the *vertical linkage*s (VL) or *footloose entrepreneur* (FE) model are used.

The necessary and sufficient condition for a non-trivial geographical dimension is therefore a minimum of three regions in non-neutral space. This makes the the study of model behaviour less easy because now it is the interplay between the economic and the geographical dimension that determines long run equilibria and their stability. In fact, there is even a third dimension because it is well known that NEG equilibria are very sensitive to the initial regional distribution of manufacturing workers who are the agents that are assumed to be interregionally mobile and sensitive to agglomeration forces. Krugman (1991) himself was the first to note that this path dependency implies that "history matters".

This paper will follow these three dimensions. First, the *economic* parameter space *E* sets out the main characteristics of economic behaviour. In all model variants the most important ones are the relative size of the manufacturing sector (δ), the elasticity of substitution (ε) and the transport costs (*T*). Second, the model size *n* and the corresponding *n*x*n* distance matrix *D* defines the *geographical* component for an *n*-region model. Third, the initial distribution of manufacturing labour over the regions $\Lambda = \{\lambda_1, \lambda_2, .., \lambda_n\}$ represents the *historical* component of the model.

[1] First nature geographical factor endowments like soil, climate etc are not considered here. The acronym NEG (New Economic Geography) covers the original core-periphery model as well as other versions. Parameter*T* sets the "iceberg"' fraction *1/T* of a transported good that melts away over one distance unit. See Fujita, Krugman, and Venables (1999) or Baldwin et al. (2003) for an overview and introduction to NEG models. Ottaviano and Robert-Nicoud (2006) discuss the main features of current classes of NEG models. Annex A summarizes the main equations of the CP and the FE model used here.

Because we want to know to what extent conclusions drawn from one economic model will be relevant for the same economic model with a different spatial structure, the analysis in this paper is essentially *geographical*: instead of applying different E's and Λ's on an identical D_0 we will examine the effect of different D's using an identical E_0 and Λ_0. This is an essential difference with the major part of the NEG literature where the simulation of many different economic parameter spaces is the rule. We can not do both at the same time but a minimum of variation of E_0 is feasable. The CP model and the FE model are analysed using two economic parameter spaces E_1 and E_2 defined as follows:

E_1 : δ=0.3, ε=3 and T=1.35
E_2 : δ=0.4, ε=4 and T=1.49

This gives four cross-combinations of {E_1, E_2} and {CP, FE} which are labelled as model CP1, CP2, FE1 and FE2. As will be illustrated in the next section, the four models not only produce quite different size distributions but also their spatial outcome is quite different. The values chosen are typical for a medium-high and a medium-low aggomeration outcome and stem from simulations with many economic parameter configurations on one and the same geography as done by Stelder (2005) and Brakman et.al. (2008). The choice for the CP model and the FE model is - admittedly - a disputable matter of taste. The CP model is the original and most basic NEG model with relatively strong agglomeration tendencies. The FE model is its main competitor as it is also simple but has an analytical solution and less strong agglomeration tendencies. As will become clear later on, despite the fact that these four models cover a relatively wide spectrum of economy topologies, their response to our geographical sensitivity analysis shows interesting similarities.

For Λ_0 only the flat initial distribution is used in which $\lambda_1 = \lambda_2 = \ldots = \lambda_n$. This "no history assumption" means that all regions are assumed to be equal in size from the start and simplifies the analysis enabling us to examine the general agglomeration tendencies of specific geographical spaces. Note that this assumption can only be used in non-neutral space because in neutral space a flat initial distribution is an immediate long term equilibrium. It's important advantage is that we do not need many model runs with random initial distributions for our analysis as is common practice for neutral geographies (Fujita, Krugman & Venables, 1999). In non-neutral space the resulting equilibria from many random runs will evidently form a normal distribution around locations that have a structural spatial aggomeration advantage. When the number of simulations goes to infinity these normal distributions will obviously converge to "spikes" on exactly the same places as the simulation with Λ_0 predicts directly (Stelder, 2005).

In order to keep the analysis tractable, we will abstract from any cross product of *E, D* and Λ such as a region-specific δ or ε, or a D-specific T such as lower transport costs for flat land relative to the cost of transport across mountains or waterways.

Section 2 defines the main classes of geographies of which the horizontal line market is the most simple form of a non-trivial geography. This geography is used in section 3 which presents the model results going from two to many regions. Section 4 repeats the analysis for two-dimensional models. Section 5 summarizes our conclusions and reflects on relevant future research issues.

2. Classes of geographies

The structure of an n-region NEG model is basically that of a network with n nodes and m connections between them. An important implicit model assumption is that agents minimize transport costs which implies that goods are traded between regions i and j along the shortest path through the network. A more precise definition of D is therefore that it holds all *shortest path distances* between the n regions given the m network connections.

The relevant types of geographies are here classified according to their eucledic dimension and the properties *neutrality* and *symmetry*. First, we will redefine spatial *neutrality* more formally as the case in which every region has the same potential P: $P_i = P_j \ \forall \ i,j$ with $P_i = 1/\Sigma_j \, d_{ij}$.

Figure 1. Classes of geographies

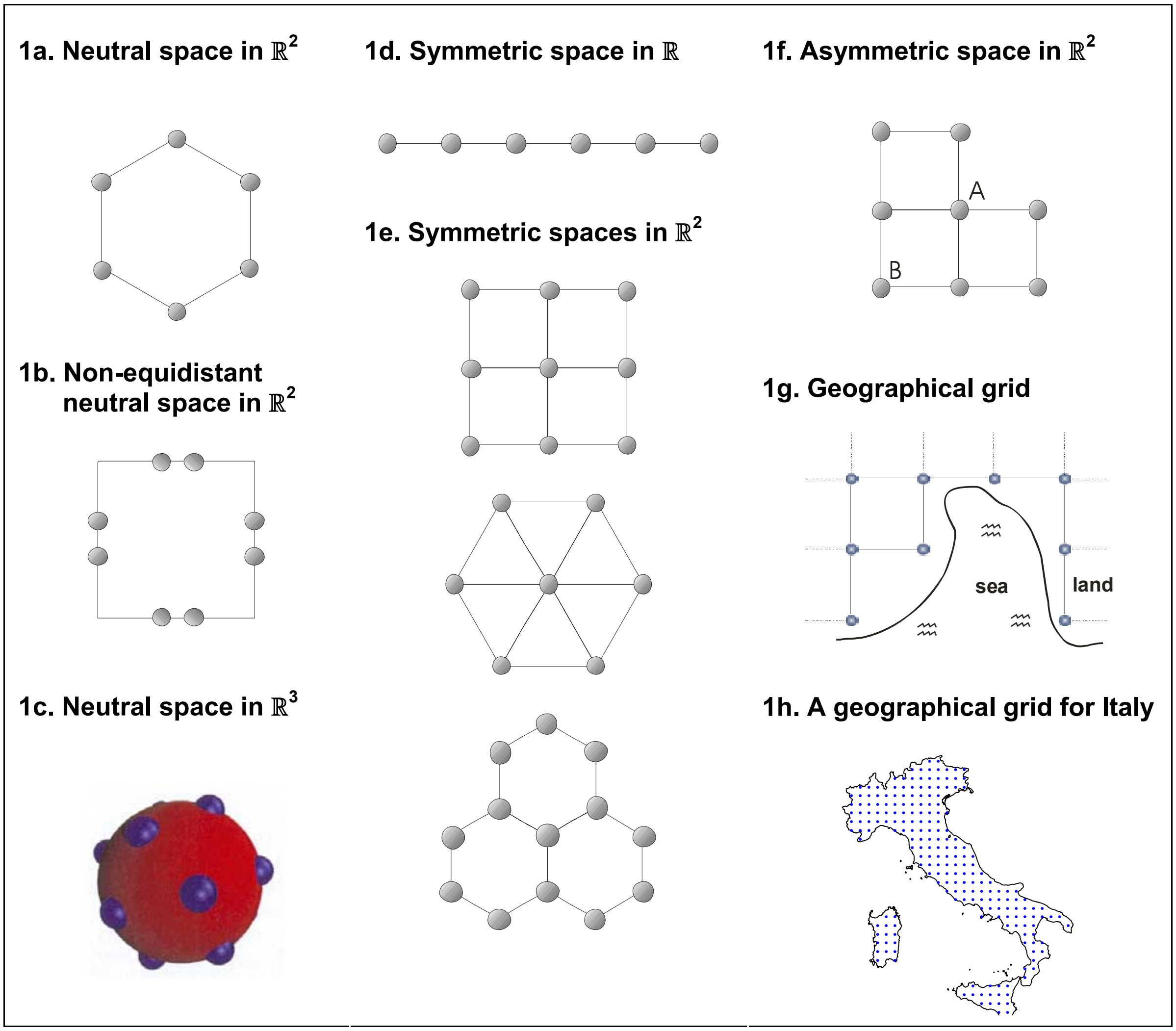


Figure 1a shows a neutral space of 6 locations on an equidistant hexagonal network in $\mathbb{R}^2$. Note that in order for the neutrality condition to hold not all connections need to be of equal distance because the example in Figure 1b is also spatially neutral. Figure 1c shows the equivalent of type 1a as an equidistant network on a globe in $\mathbb{R}^3$.

Next, the simplest introduction of non-neutral space is to delete one connection in Figure 1a which cuts the hexagon into a line segment congruent to a straight line in $\mathbb{R}$ (Figure 1d). The assumed equal distances between all direct neighbouring locations leads to a special property which we will here define as *symmetry*. The symmetry condition holds when $\forall i$ with P_i < Max (P_i) $\exists j \neq i$ for which $P_i = P_j$. Due to this property, model 1d is a discrete approximation of a Hotelling continuous line market where two identical firms can hold spatially mirrored equal market shares. Section 3 and 4 will show that the (non)existence of symmetry is important because it has a dominant influence on the urban distribution in the long term equilibrium.

In the same way, the continuous economic plane in $\mathbb{R}^2$ can be approximated by symmetric networks as depicted in Figure 1e, of which the hexagonal example at the bottom is the familiar urban landscape of central places. The crucial difference with the world of Christaller and Lösch, however, is their boundedness because in any NEG model $n<\infty$. Finally, we enter the world of real geographies when symmetry no longer exists and the network can be of any form. In

Figure 1f only one location of the square grid in Figure 1e is deleted which gives location *B* a unique value P_b lower than the highest potential value P_a. If more points are deleted according to the geographical shape of a country (Figure 1g) we obtain the geographical grids as introduced by Stelder (2005). Figure 1h shows a geographical grid for Italy.

For the purpose of this paper it is important to stress an essential difference between an asymmetric geography like Figure 1h and a symmetric geography like the horizontal line in Figure 1d. This is related to how we want to interpret model size. Starting with a two-regional model A, if in a larger model B we increase the number of regions from 2 to 4 by simply adding an extra region at the left and the right at the same distance as the distance between the original two regions, the line "becomes longer" and $D_b(1,2)=D_b(2,3)=D_b(3,4)=1$ if $D_a(1,2)=1$. The other interpretation would be that for another larger model C we keep the size of the line in model A constant at 1 and redevide this distance over four regions in stead of two. The line then "gets a higher resolution" and $D_c(1,2)=D_c(2,3)=D_c(3,4)=0.33$. These two interpretations will obviously lead to different results but model C can simply be rescaled back to model B by adjusting the transport parameter T in equation (3) of Appendix A so that transport costs T_{rs} between two neighbouring locations r and s are the same in B and C. In our vocabulary: combined with a specific economic parameter space E_{b0} and E_{c0} the models using D_b and D_c will behave identically when n increases. In section 3 therefore only the simple first interpretation of model B will be used.

For asymmetric models, however, it does make a difference whether we increase the size or the detail of the model. Let us take a land-locked country like Austria as an example in analogy to the geographical grid of Italy in figure 1h. Increasing the size according to the first interpretation would imply extending it with parts of its direct neighbours Germany, Switzerland, Italy, Slovenia, Hungary, Slovakia and Czechia. This would introduce international trade in the model while the second interpretation would only model Austria itself in more geographical detail.

It is beyond the scope of this paper to examine both options. The response of the model to higher resolutions and the related question of how much geographical detail is relevant for economic agglomeration analysis is an issue in itself and will be examined in future work. Therefore in this paper only the first interpretation will be used: increasing the size of the model by adding extra regions while keeping the geographical resolution constant.

In section 3 the influence of geographical size on model behavior is analyzed for geographies of type 1d. In section 4 the analysis is repeated for two-dimensional geographies of type 1e and 1f. Section 5 concludes and reflects on relevant future work.

3. The influence of geographical size in one-dimensional space

In this section we will restrict ourselves to the simplest form of a non-neutral geography: a one-dimensional space on a horizontal line market as depicted in figure 1d.

The models were tested for the range *n=3 to 100* using the initial flat distribution Λ_0 with $\lambda_1 = \lambda_2 = ... = \lambda_n$. Due to the non-neutral geography some regions have better access to competing markets than others which leads to a long term equilibrium with $m<n$ agglomerations where all manufacturing labor has become concentrated ("*cities")* and (n-m) remaining locations from which all manufacturing labor has moved away and have only agricultural labor left ("*villages*").

As an example of the full range of all 97x4 simulations for all four models Figure 2-4 show the location and size of the cities in the long term equilibrium for n=22, n=23 and n=93 respectively.

Figure 2. Long term equilibria for a horizontal line model with 22 locations
Regional shares of national manufacturing labor

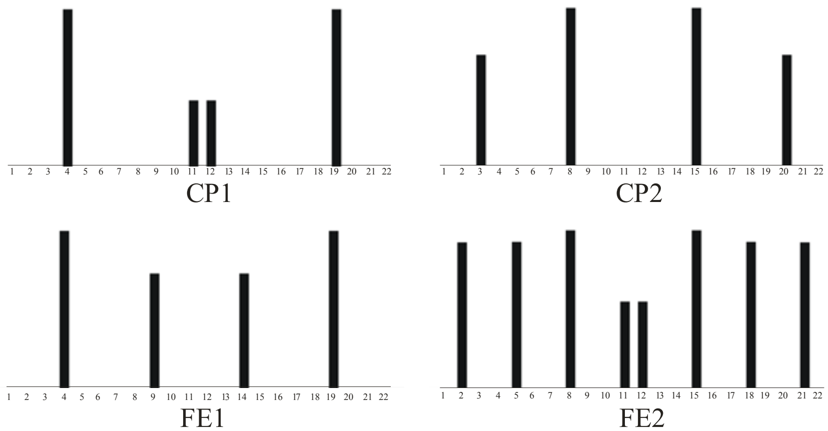


Figure 3. Long term equilibria for a horizontal line model with 23 locations
Regional shares of national manufacturing labor

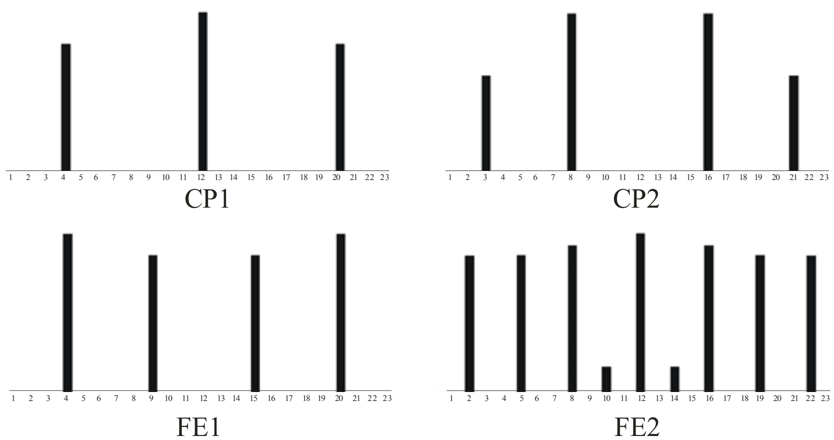
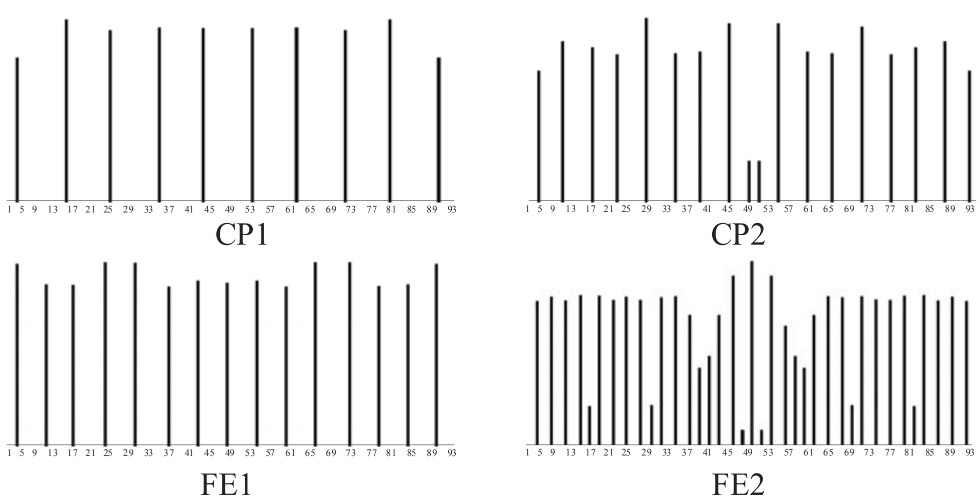


Figure 4. Long term equilibria for a horizontal line model with 93 locations
Regional shares of national manufacturing labor



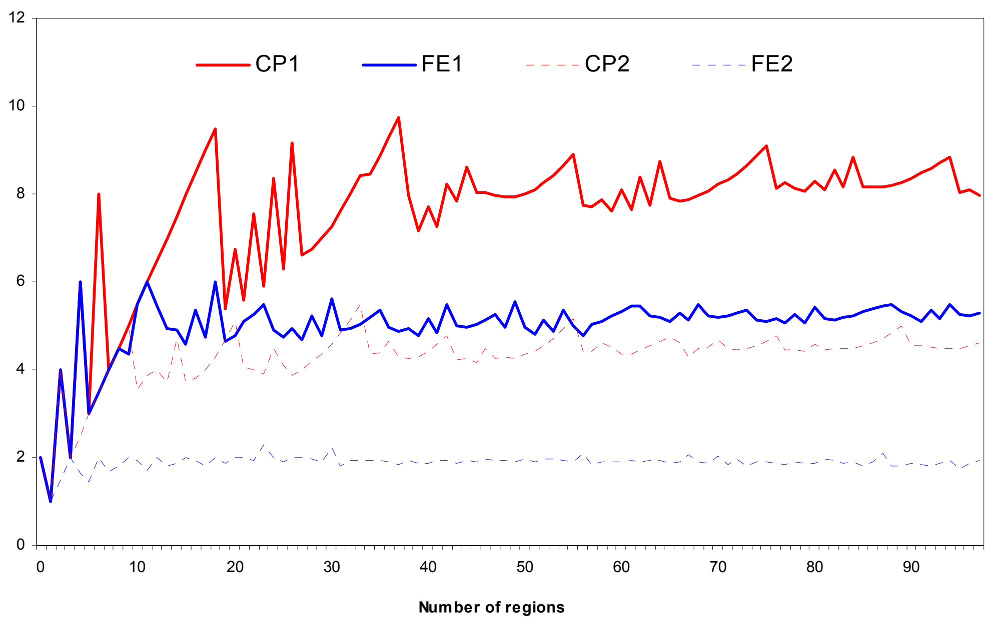
The comparison between figure 2 and 3 reveals how sensitive small models are when going from even to uneven numbers. The CP1 model predicts two small second order cities in the centre when *n=22* but one large central agglomeration when n=23. The central market captured by location 12 in the latter case has to be shared between location 11 and 12 in the first case who have a mirrored but equally competitive spatial market power. The same type of shift happens for model FE2. Figure 4 shows how the predicted city structure becomes more differentiated for larger models.

The model simulations have been examined with respect to their size distribution and locational structure. First, in order to measure the model behaviour as *n* increases, we need a slightly modified Herfindahl index that is invariant to *n* for otherwise identical urban hierarchies. This *Urban Herfindahl Index (UHI)* is introduced in Appendix A. Figure 5 gives the *UHI* of CP1, FE1, CP2 and FE2 for *n* increasing from 3 to 100. The effect of going from even to uneven numbers is clear from the very volatile results for small values of *n.* The *UHI* jumps from high values for *n*=3,5,7,9 to low values for *n*=4,6,8,10.

This effect becomes less important for larger values of *n*, although CP1, FE1 and CP2 remain to show substantial fluctuations going from *n* to *n+1*. The most stable model is FE2 that has an almost constant *UHI* for *n*>20. The *UHI* for CP2 and FE1 also seems to converge to a constant value, but not untill *n* has become 60 or larger. As expected, CP1 and FE1 have a significantly higher centripetal tendency compared with CP2 and FE2 because higher values for σ and τ lead to more spreading[2].

These results are a strong indication that analytical or numerical results from small models can not be generalized to larger systems. Apparently the “degrees of freedom” for cities to grow or decline increase as the model expands. Figure 2 suggests that a minimum of 20 (FE2) to 50 (CP1, CP2 and FE1) locations is necessary to derive conclusions from a NEG model that are robust to model size.

Figure 5. Urban Herfindahl Index for a horizontal line with increasing size



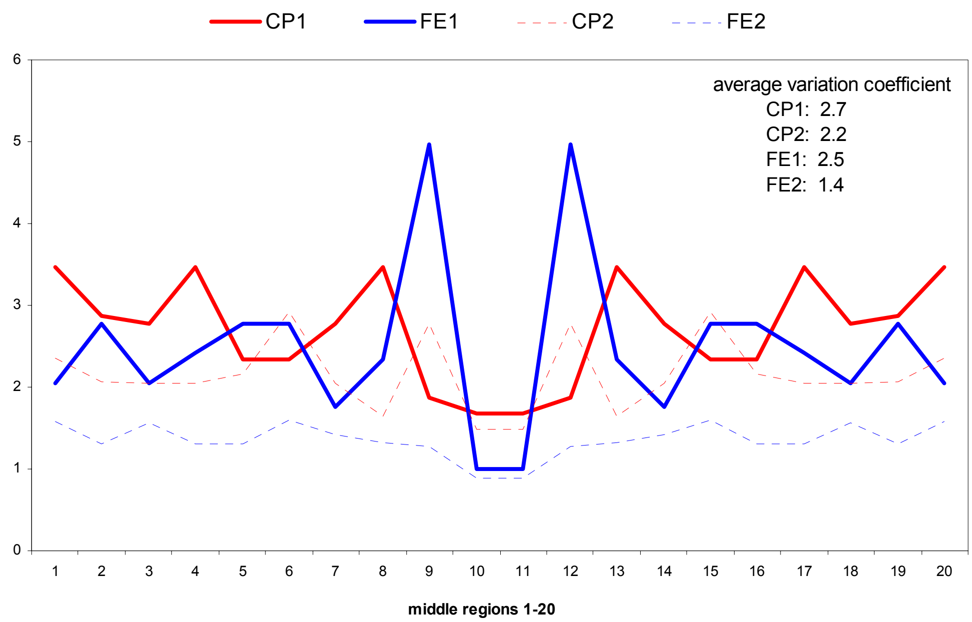
[2] The higher value of δ in CP2 and FE2 works in the opposite direction, but this was done because keeping δ at 0.3 made the centripetal forces too low compared with CP1 and FE1.

This is, however, yet half of the story. The models do not only predict the size of the cities but also their location. To what extent is the predicted spatial distribution invariant to *n*? Figure 2-3 already showed that cities are predicted at different places in models of different size and the interested reader can comes to his own intuitive conclusion by downloading a full visual presentation of all 4x97 simulations for the four models[3]. But how do we systematically compare a 20-region model with a 40-region model in this respect? The results discussed above suggest that we should skip the smaller values of *n*. We can then check whether the stability found in Figure 5 of the size distribution for models larger than 50 regions is also valid for the spatial distribution.

An impression of the spatial stability can be achieved by selecting the 20 middle regions of all models for the even values of *n* with 50 < *n* < 100. That is, we start with the model result of 50 locations and monitor the size variation of the 20 middle regions for *n* = {50, 52,..., 100}. Every increase from *n* to *n*+2 is then interpreted as an extension of one region at the left and right end of the horizontal line and we evaluate how this affects the spatial distribution of the original 20 middle regions. In order to correct for scale effects of larger systems the size distribution of the 20 middle regions is rescaled to a total population of one for all values of *n.* After this rescaling, the variation of the population of each of these 20 locations can be calculated over the 25 models of increasing size (n=50,52,..,100). The results are shown in Figure 6.

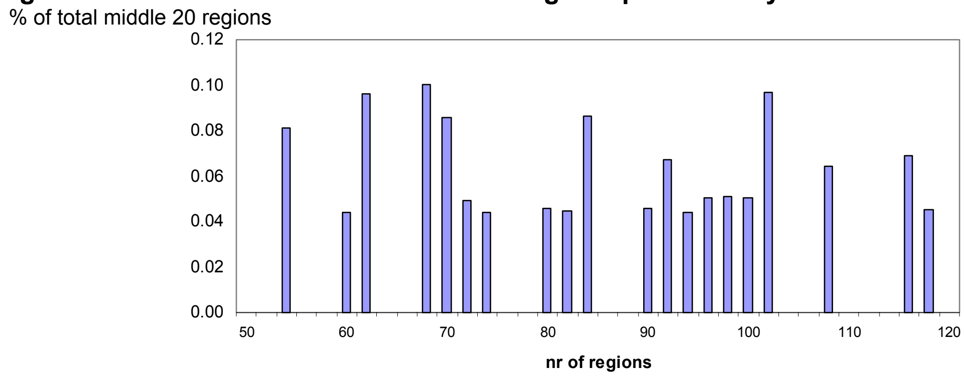
On average the order of stability (FE2 > CP2 > FE1 > CP1) is the same as in Figure 1 but CP1, CP2 and FE1 show very mixed patterns for individual locations. Over the model range 50<*n*<100 FE1 predicts very different values for region 9 and 12 but is relatively stable for the most centered regions 10 and 11. Moreover, we should be aware that the lowest variation coeffient found (0,89 for region 10 in model FE2) is still very high[4]. Figure 7 illustrates this by showing the predicted size of region 10 in model FE2 for n = {50, 52,..., 100}. In 10 out of all 25 models region 10 gets no value at all and the non-zero values are very volatile for *n*. The predicted value for region 10 seemed to become stable for *n*>90, but this was checked by extending the range of simulations to n=120. Clearly, in the range 100 < *n* < 120 region 10 becomes unstable again.

Figure 6. Variation coefficient for the 20 middle regions of models with 50 < n(even) < 100



[3] Find the relevant presentation at [to be added]
[4] A value of 0.5 means that the average absolute deviation from the mean is 50%.

Figure 7. Size of the 10th of the middle 20 regions predicted by model FE2



4. Increasing model size in two-dimensional space

The stability analysis was repeated for two-dimensional symmetric geographies of type 1e in Figure 1 using an *m*m* square grid for *m*={3, 4, .., 30} which makes the increase of the model size quadratic following *n* = {9, 16, 32,..., 900}. The results are given in Figure 8 and lead to two conclusions.

First, from *n* = 9 to 225 we can see the returning pattern of alternating high and low agglomeration when changing from even to uneven numbers. This corresponds with the fluctuating values of the *UHI* for *n* = 1 to 25 in Figure 5. Apparently this "geographical" effect is more dominant in the twodimensional case because CP1 and FE1 behave exactly the same over the range *n*=9 to 144 (*m*=1 to 12), while in Figure 5 this is only the case for *n(m)* =1 to 8 .

Second, Figure 8 shows more or less the same stability properties of the four models but now convergence to stability only starts to appear for much larger models than in the one-dimensional case of Figure 5. FE2 becomes stable for n>100. For the other three convergence (if any at all) should not be expected for values of *n* < 1000. The intuitive explanation of this different behaviour is that agglomeration forces have more degrees of freedom in two-dimensional space than on a horizontal line. Because centripetal and centrifugal forces have their influence on manufacturing labor in other locations in all directions, more spatial hierarchy configurations become possible. The stable value of the *UHI* to which the FE2 model in Figure 8 converges is 19, which is ten times higher than the stable value of 1,9 found in Figure 5.

Figure 8. Urban Herfindahl Index for a square grid with increasing size

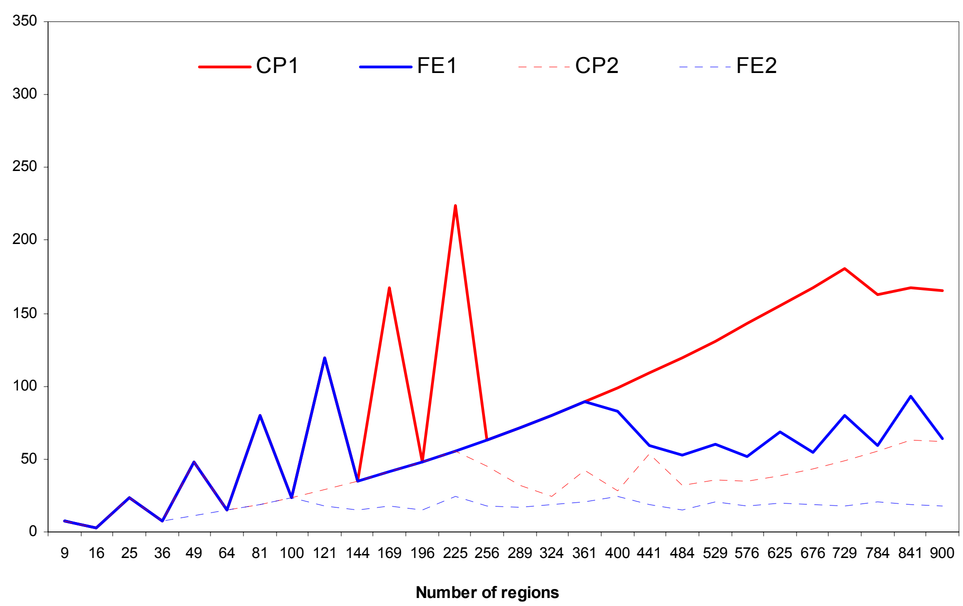
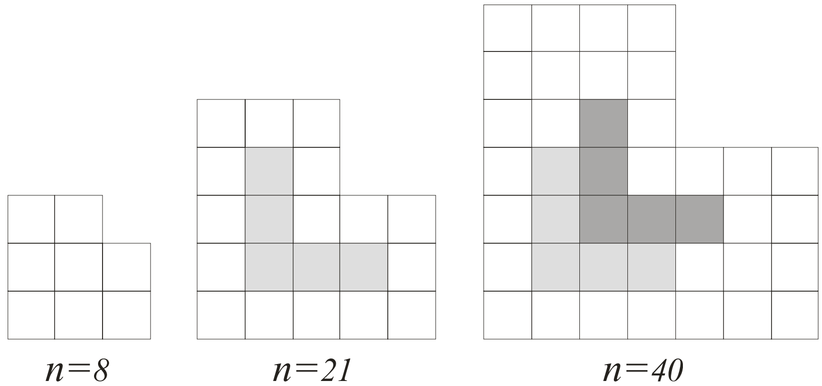


Figure 9. Increasing the size of an asymmetric model of type 1f



How do the results look like when asymmetry is entered in this model? As discussed in section 2, a model of type 1f in Figure 1 is the most simple form of an asymmetric two-dimensional space. Figure 9 shows how increasing size is interpreted in this case. For *m=3* the upper right location is deleted which makes *n=8.* Next, for *m=5* the three upper right locations are removed which makes *n*=21 etc. As in the previous case a series of 30 models was created for *m*={3, 5, 7, ..., 57} which now in total size go up to the largest model of 2465 locations.

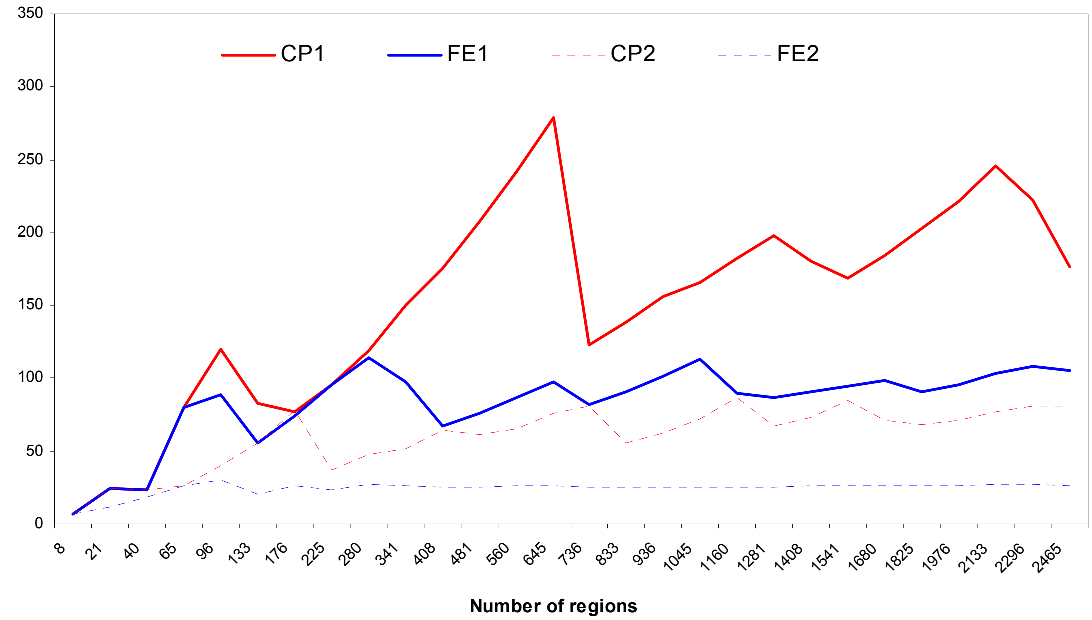
The results given in Figure 10 show approximately the same fluctuations over the range *n=400 to 900* as in Figure 8*.* Note that the steep jumps in the range of small models due to even and uneven numbers are now disappeared because *m* only increases over uneven numbers. FE2 is again by far the most stable model but on average all UHI values are 40% or more higher than in the symmetric case (see table 1). The size range where the models start to become stable is also higher. FE2 reaches stability around *n*=200 and for FE1 and CP2 stability, or at least fluctuations around some stable level, starts to emerge beyond *n*=1500. This did not show up in Figure 8 were the 30 simulations did not go beyond *n*=900. CP1 shows no stability at all.

An exploratory analysis revealed that the stability of the locational structure in model 1e and 1f is just as weak as was shown for the horizontal line model in Figure 6-7. For model 1e the stability of the center 4x4 block of locations was examined for the ten square *mxm* models with *m*={12,14,..,30}. The average variation coefficients found were about 20% lower than the values mentioned in Figure 6 but this may be related to the short series of ten size expansions in which some of the 16 locations in the centre 4x4 block remain zero in all model sizes. Expanding the series to *m*=60 or more should be required but goes beyond our present computer capabilities. The exact measurement of locational stability is questionable for the asymmetric model in figure 9. In a symmetric model we can take the middle 20 regions on a line or the 4x4 middle block of a square grid but which part of the models in Figure 9 should we take? The grey shaded block in the model with *n*=21 seems logical but has no one-to-one relation with either one of the two grey blocks in the *n*=40 model. For the light grey area variation coefficients of roughly the same size were found as for the square model.

Table 1. Average value of the Urban Herfindahl Index
Average over the 15 largest models of Figure 8 and Figure 10

Model	*CP1*	*CP2*	*FE1*	*FE2*
a. symmetric type 1e (fig 8)	132	42	69	19
b. asymmetric type 1f (fig 10)	183	74	96	26
ratio b/a	1.4	1.8	1.4	1.4

Figure 10. Urban Herfindahl Index for an asymmetric grid with increasing size



5. Concluding remarks

In the literature analytical results for NEG models are mainly achieved for two-region models. The computer simulations presented in this paper show what is logical to expect but difficult - if not impossible - to derive analytically: multiregional models behave different from the two-region model and large models behave different from small models. The extent to which they do depends on the economic structure of the model and their parameter values. The results in this paper clearly indicate that model configurations with a high agglomeration tendency are less robust to model size and the CP model leads to much more volatile results than the FE model. The latter can be configured to a high level of invariance to model size, but even in a perfectly homogeneous geographical space like a horizontal line stability only starts to appear when the model has 20 regions or more. In two-dimensional space this 'stability threshold' rises to 100 and when a minimum of asymmetry is introduced the minimum number of regions becomes 200. Other model configurations with more centripetal forces do not converge to any stability in two-dimensional space or only when the models become very large.

These results suggest that we should carefully design our models when real geographies are implemented. The decision on whether we should model a country at state, province, county or city level, or with or without foreign trading partners, usually depends on issues like data availability or the assumed relevance of geographical detail and international trade. This analysis shows that the choice of the number of regions is an issue in itself. Simple two-dimensional spaces with less than 50 regions should be avoided. Real geographies should preferably have a lot more.

Appendix A. The main equations of the CP model and the FE model

Following the standard notation of Fujita, Krugman & Venables (1999) and Brakman, Garretsen & van Marrewijck (2008) the core equations of the CP model are:

(1) $y_r = \delta\lambda_r w_r + (1-\delta)\phi_r$

(2) $I_r = \left[\sum_{s=1}^{R} \lambda_s T_{rs}^{1-\varepsilon} w_s^{1-\varepsilon}\right]^{1/(1-\varepsilon)}$

(3) $T_{rs} = T^{D(r,s)}$

(4) $w_r = \left[\sum_{s=1}^{R} Y_r T_{rs}^{1-\varepsilon} I_r^{\varepsilon-1}\right]^{1/\varepsilon}$

In (1) for every region r y_r is total income, δ is the fraction of income spent on manufacturing goods, w_r is manufacturing wages, λ_r is the regional fraction of national manufacturing employment and φ_r is the regional frcation of national agricultural employment. Next, in (2) I_r is the regional price index and T_{rs} is the iceberg transport costs indicating the number of goods needed to be shipped from region r in order to have one unit of goods arriving in region s. Equation (3) shows that this is equal to the transport cost parameter T in the case of a two-region model with $D(1,2) = 1$ and raised to the power $D(r,s)$ in an n-region model. Finally, in (2) and (4) ε is the elasticity of substitution derived as $\varepsilon = 1/(1 - \rho)$ with ρ being the substitution parameter in the aggregate utilty function.

System (1)-(4) determines the short term equilibrium value of w_r given the parameter values for δ, ε and T space (this is the economic parameter space E_0 mentioned in the paper) the and the inital values of λ (Λ_0).

One of the assumptions behind the core equations of the CP model is the production function for manufacturing (M) firms in which manufacturing labor is the only product factor with increasing returns to scale. Following Robert-Nicoud (2005) this defines total costs C for the typical M-firm producing $x(i)$ quantities of manufacturing good i as

(5) $C(x(i)) = w_m\,[F + \beta x(i)]$

with F and β as the fixed and variable costs parameters[5] and w_m indicating the wage in the manufacturing sector. It is in this equation where the CP model differs from the footloose entrepreneur model (FE) which uses

(6) $C(x(i)) = w_m\,F + w_a\,\beta x(i)]$

where w_a is the wage in the agricultural sector. The production function (6) has fix costs of skilled (manufacturing) labor and variable costs of unskilled (agricultural) labor (Forslid & Ottaviano, 2003). This makes the equations (2) and (4) no longer recursive because in the FE model the price index equation (2) simplifies to:

(7) $I_r = \left[\sum_{s=1}^{R} \lambda_s T_{rs}^{1-\varepsilon}\right]^{1/(1-\varepsilon)}$

[5] These parameters do not show up in (1)-(4) due to some parameter normalizations. See Brakman et. al. (2008) chapter 4 for details.

The FE model equations (1), (3), (4) and (7) enable an analytic solution in stead of the numerical solution that is needed in the CP model. However, as any programmer will quickly find out, the reduced form equation for the wages in the two-region case as presented in Brakman et. al. (2008) or Forslid and Ottaviano (2003) becomes too long and cumbersome for 3 or more regions so numerical techniques are needed after all to find the analytical solution.

Finally, the determining of the long term equilibrium is the same for both models. Given the short term equilibrium for w_r the model assumes that manufacturing workers migrate to regions with the highest real wage ω_r according to

(8) $\omega_r = W_r I_r^{-\delta}$

and a migration reaction relative to the regional deviation from the average national real wage $\acute{\omega}$:

(9) $\lambda_{r,t+1} = \eta \lambda_{r,t} \omega_r / \acute{\omega}$

Here η is a migration sensitivity parameter and index t indicates the iteration number. The model simulation stops when real wages are equalized across the regions and convergence is reached to the long term equilibrium Λ_{t^*} when for $\forall r \ \lambda_{r,t^*+1} / \lambda_{r,t^*} < (1+ \kappa)$[6].

[6] In all simulations used in this paper η is set to 1 and the break-off condition κ is set to 0.0001.

Appendix B. An Urban Herfindahl Index for comparing urban hierarchies of different size

Let an urban structure $U = \{u_1,..,u_N\}$ be defined by $u_i = p_i / \Sigma_i\, p_i$ with p being population or any other absolute size indicator. Then the *Herfindahl Index* HI = $\Sigma_i\, (u_i)^2$ can be interpreted as a measure of agglomeration ranging from 1 (full agglomeration into 1 city) to 1/N (no agglomeration: all cities are of equal size). Because the minimum of HI depends on N, two distributions U_1 and U_2 can not be compared unless $N_1 = N_2$. The *Normalized Herfindahl Index* NHI solves this problem by subtracting the minimum 1/N and rescaling the interval back to (0,1) with NHI = 1/(1-1/N) (HI -1/N).

The correct interpretation of the NHI, however, is that it really measures market concentration for different numbers of firms. If one firm achieves 75% of a market with just one competitor taking the remaining 25%, market concentration is considered to be higher than in a 4-firm situation where two pairs of firms take each 37,5% and 12,5%. Table 1 shows that the NHI declines from 0,250 to 0,083, and further down to 0,036 in the comparable 8-firm situation.

If a distribution is interpreted as an urban hierarchy, however, the NHI does not give identical values for identical agglomeration structures. Figure A1 shows the three distributions of table 1 on a horizontal line market with locations at a constant distance from each other. The distinctive property of this urban hierarchy is an alternating pattern of a first order city and a second order city at distance 1. A correct urban agglomeration index would have to be invariant to whether we define the urban hierarchy over locations 1-2, 1-4 or 1-8. This is achieved by deleting the rescaling of the maximum value to 1 in NHI which gives us an *Urban Herfindahl Index* UHI = N (HI -1/N). In this example, the UHI has a value of 0,250 invariant to N=2, N=4 or N=8.

As the NHI, the UHI has a minimum of zero when all cities are of the same size but its maximum is N-1. This follows our intuitive interpretation: suppose we start to travel from a large city at location 1 to the right along the horizontal line. Then, the more "empty" locations[7] we pass, the more impressive the agglomeration at location 1 becomes. If all locations 2-8 are empty, the UHI over the whole space covering 1-8 is 7. The interpretation of this maximum is straightforward: one player has become a monopolist winning a game with 7 other players. Contrary to the NHI which is always 1 for monopoly, the UHI indicates that winning a game with more players makes the winner more competitive.

A normal application of the UHI will be to compare two urban size distributions U_1 and U_2 with $N_1 \neq N_2$ regardless of their spatial pattern. Table A1 shows that alternatives like the Zipf coefficient β or Gini coefficient γ have the same problem as the NHI[8]. An identical UHI will only imply an identical β or γ when N_1=N_2.

[7] An "empty" city i with $u_i = 0$ is comparable with a firm with a zero market share. Such a firm exists selling other products or selling the product on other markets. Empty cities play a role in New Economic Geography models as locations with agricultural employment, but with a zero share in national manufacturing employment.

[8] The standard Zipf regression equation is $log(R_i) = \alpha - \beta\, log(P_i)$ with ranking number R_i and size P_i for i=1..N. The coefficient β changes with N unless the urban hierarchy follows the pure Zipf distribution with β=1. As N increases, the cumulative distribution of U converges to a continuous curve which makes the Gini coefficient γ also N-dependent.

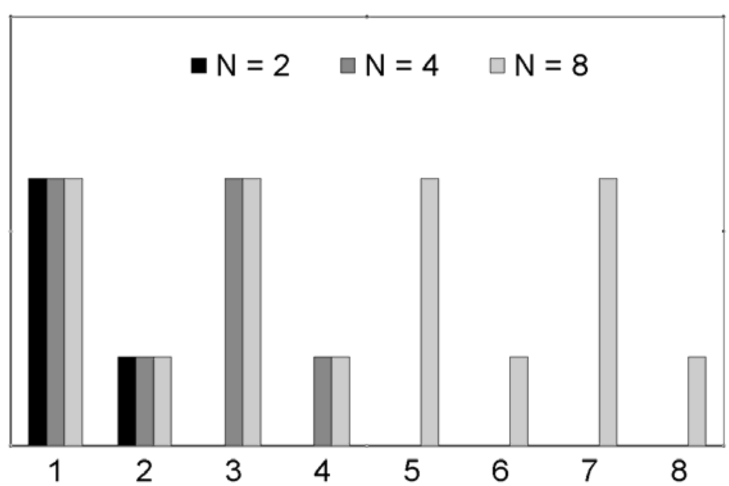
Table B1. Measures of concentration in markets of different size

% market share

	N	2	4	8
city/firm	1	75.0	37.5	18.8
	2	25.0	12.5	6.3
	3		37.5	18.8
	4		12.5	6.3
	5			18.8
	6			6.3
	7			18.8
	8			6.3
	HI	0.625	0.313	0.156
	NHI	0.250	0.083	0.036
	UHI	0.250	0.250	0.250
	Zipf β	1.585	0.908	0.838
	Gini γ	0.500	0.333	0.286

Figure B1. Identical agglomeration in systems of different size

Absolute population size on a horizontal line market



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