Simultaneous choice of process and product innovation when consumers have a preference for product variety

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Abstract

This paper analyses simultaneous product and process innovation if demand is characterized by preference for product variety. It investigates the strategic decisions of two identical duopolists, who choose production technology as well as product differentiation through R&D investment and discusses factors which determine the optimal proportion of R&D activities. If firms coordinate their R&D activities and either conduct R&D in one lab, or R&D-cost functions are sufficiently steep, they will invest more in R&D than under competition and shift the optimal proportion of R&D investment towards product innovation. Firms’ investment is also driven to product innovation if consumers’ willingness to pay is high.

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1. Introduction

Empirical evidence indicates that firms usually have a portfolio of R&D projects, some targeted more at process innovation, some at product innovation. The management literature mentions ratios like 60:40 of total R&D budget for process relative to product innovation in Japanese firms. In fact, it is frequently discussed that Japanese firms invest more in process innovation while Western firms concentrate more on product innovation. As possible reasons for differing innovation strategies, cultural distinctions, differences in patent policy as...
well as differences in firm size are specified. But these explanations fail whenever heterogeneous R&D portfolios are observed among firms of similar size within one country. While several empirical studies observe changing R&D portfolios over time for several industries, others point to the central role of strategic interaction and interfirm cooperation in shaping firms’ R&D portfolios. These observations lead to the hypothesis that the magnitude of potential market demand as well as the strategic framework also influence firms’ innovation strategies. The theoretical model presented here analyzes these two alternative explanations, and, while the results confirm most empirical findings, derives conditions which also help to understand contradictory empirical evidence.

Empirical observation allows at least for the vague conclusion that firms choose strategically among the two alternative kinds of innovation, usually without a complete specialization in one. Until recently, this fact was widely ignored in the microeconomic theory of innovation while it has already been investigated in the business literature: the “technological life-cycle” model of Abernathy and Utterback (1982) pictures the absolute frequency of product and process innovation in a product line and its associated production process. Initially, when market needs for a new technology are ill-defined but market potential is large, product innovation will tend to be predominant. The emphasis will change from product to process innovation when performance criteria have been standardized and prices become the new critical factor of success.

This concept of a product life-cycle was recently translated into a formal model by Klepper (1996), who depicts industry evolution and identifies the role of firm innovative capabilities and size in conditioning R&D spending. Yin and Zuscovitch (1998) extend Klepper’s analysis by considering strategic effects. They find that the “composition of firm’s R&D portfolios (…) depends on the firm’s initial market share and on the subsequent effects of R&D on the post-innovation market structure,” and that therefore, a large firm invests less in product and more in process innovation than a small firm. In contrast to the model presented here they consider an ex ante asymmetric setting in which the different incentives to introduce a drastic product innovation are analyzed.

Bonanno and Haworth (1998) consider a vertically differentiated industry and compare the choice of a firm between either product or process innovation under Cournot and Bertrand competition. They confine their analysis to the decision of a single firm while the present

3 See Albach (1994). Cultural reasons for this difference are discussed by distinguishing between the process-orientation in Japan originating from the Samurai tradition and the result-orientation in America and Europe originating from Calvinistic moral values. Scherer (1991), Klepper (1996), Yin and Zuscovitch (1998) suggested the effects of different firm sizes as possible explanations. Eswaran and Gallini (1996) examine the role of patent policy in redirecting the mix of product and process innovation towards a more efficient technological change.


5 Pine et al. (1993) document that more and more firms structure their organization in order to allow them to engage in both kinds of innovative activities simultaneously: incremental product as well as process innovations.

6 Most models on innovation suggest that firms invest either in process innovation or product innovation and determine optimal innovation strategies under this assumption. See Reinganum (1983), De Bondt (1997) for comprehensive surveys of innovation models.
paper extends the analysis to two firms, choosing two variables simultaneously, marginal costs and product characteristics.

We consider innovative activity within a given product life-cycle, and thus, focus on the development of a market after it has been pioneered. Process innovation is understood as a lowering of marginal production costs and product innovation as a reduction of product substitutability. Our distinction between product and process innovation is much in line with Eswaran and Gallini (1996), and is thus, “similar to that made between ‘horizontal’ and ‘vertical’ innovation, respectively (…)”. While process innovation can be viewed as ‘vertical’ improvements in the product itself (i.e. it may be an increase in the effectiveness of a drug, such that consumers require only a fraction of the previous dosage), product innovation as described in our model is horizontal: the innovation is not superior in absolute sense but is simply different and increases consumers’ valuation of the product because of preferences for distinct varieties (i.e. the development of different forms to administer a certain drug).

In this strategic setting, it can be shown that, in accordance with most of the research based on Abernathy and Utterback (1982), an increase in consumers’ willingness to pay causes firms to increase R&D investment but also to shift their R&D investment more to product innovation if the R&D efficiencies of the two kinds of innovation are similar. Obviously, the firms have a lower incentive to reduce their costs to practice a tougher price competition when the market size increases.

Another question addressed in this paper concerns the effects of cooperative R&D activities on the optimal proportion of innovative activities. Two different forms of cooperative agreements are analyzed. The case of R&D-cartel is compared to an RJV-cartel, assuming that the latter allows firms not only to coordinate R&D strategies as to maximize joint profits but also to share R&D efforts. It can be shown, that firms invest more in an RJV-cartel than under competition and that investment is shifted towards product innovation. In this setting, the increase in efficiency (no wasteful duplication of efforts) overcompensates the impact of negative externalities on firms’ investment incentive. If the cooperative agreement is limited to pure joint profit maximization, the effect on innovation strategies depends strongly on the assumptions concerning the underlying R&D-cost functions. Still, it is most probable that firms will shift their investment towards product innovation. In fact, these results help to explain the empirical findings of Khanna (1995), Lante and Speckman (1997), Karlsson (1997).

We find that product innovations exhibit positive externalities if consumers have preferences for product variety. In relation to the findings of D’Aspremont and Jacquemin (1988) it can be shown that this positive competitive spillover on the product market can (under certain conditions concerning the R&D-cost functions) outweigh negative externalities from process innovation. It can therefore take the role of technological spillovers and cause welfare improvements even in case of cooperative decision making without cost sharing. This finding qualifies previous findings on welfare effects of cooperative R&D and allows us to call for a more favorable treatment of research joint ventures than implied by solely taking

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7 A large proportion of the economic gains from innovative activity in the last century can be attributed to incremental innovations, as Kline and Rosenberg (1986) argue.

into account technological spillovers. If cooperative agreements on R&D include horizontal product innovations in an existing product line they tend to be socially beneficial.

The following section describes the demand structure of a simple differentiated duopoly. Section 3 derives firms’ quantity decisions for any level of marginal costs and product substitutability. We characterize optimal investment behavior under R&D competition in Section 4 and compare this to investment under R&D cooperation in Sections 5.1 and 5.2. Comparative static properties in the size of the market are presented in Section 6 and welfare implications are analyzed in Section 7. The last section provides some concluding remarks.

2. The model

On the demand side, we assume that consumers have a preference for product variety. They buy all products but consider them to be more or less substitutable to each other. For simplicity, we assume that firms \( i, j \) face linear inverse demand functions of the following form:

\[
P_i(x_i, x_j, \delta_i) = a - (x_i + \delta x_j),
\]

where \( a > \max\{c_i, c_j\} \) with \( c_i, c_j \) representing firms’ marginal production costs and \( \delta \in [0, 1] \) measures the degree of product substitutability. The higher \( \delta \), the higher is the degree of substitutability. When \( \delta \) tends to zero, firms effectively become monopolists, while for \( \delta = 1 \) the good of firm \( i \) is a perfect substitute to the good of firm \( j \).

On the supply side, we consider a duopolistic industry, consisting of two firms \( i, j \) that produce quantities \( x_i \) and \( x_j \). The two firms operate under constant returns to scale. Firms’ unit costs of production are given by \( c_i \) and \( c_j \) with \( c_i, c_j \in [0, a] \), which can be chosen through R&D investment before the market opens. The product characteristics which determine the degree of product substitutability given by \( \delta := \delta_i + \delta_j \) can also be influenced by the firms \( i, j \) through R&D investment in \( \delta_i \) and \( \delta_j \) respectively, with \( \delta_i, \delta_j \in [0, 1/2] \).

The cost function for R&D is the same for both firms and is described by \( K(c_i) + G(\delta_i) \) with \( K' < 0, G' < 0 \) and \( K'' > 0, G'' > 0 \). The higher the marginal costs and the higher product substitutability the lower is the needed research investment. We assume non-drastic innovations in the sense that there exists an initial level of costs \( c^0 \leq a \) with \( K(c^0) = 0 \) for all \( c_i \geq c^0 \) and \( \lim_{\delta_i \to c^0} K(c_i) = 0 \) and an initial level of product differentiation \( \delta_i^0 \leq 1/2 \) with \( G(\delta_i^0) = 0 \) for all \( \delta_i \geq \delta_i^0 \) and \( \lim_{\delta_i \to \delta_i^0} G(\delta_i) = 0 \). Further (to guarantee interior

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9 The main difference between this model of monopolistic competition and other models of product differentiation (horizontal differentiation models as originally introduced by Hotelling, 1929 as well as vertical differentiation models see Gabszewics and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983) is that here there is no heterogeneity in taste (there exists a single representative consumer) and that the consumer consumes a bit of every available good instead of consuming only his most preferred product. Therefore, in this model there is no asymmetry in the substitutability of various products in the industry which simplifies the analysis substantially. In models of vertical or horizontal product differentiation a product competes more with some products (its close neighbors in the product space) than with others.

10 The underlying demand structure is adopted from Dixit and Stiglitz (1977) and Spence (1976). The representative consumer’s utility is a function of consumption of the two goods and the numeraire good \( m \). It may be given by \( U(x_i, x_j, \delta) + m \) with \( U(x_i, x_j, \delta) = a(x_i + x_j) - (x_i^2 + 2\delta x_i x_j + x_j^2)/2 \).
solutions), we impose that \( \lim_{c_i \to 0} K(c_i) = \infty \) as well as \( \lim_{\delta_i \to 0} G(\delta_i) = \infty \), and we assume that no technological spillovers exist.

Firms play a non-cooperative two-stage game under complete information. In the first stage, they decide on their marginal costs by investing in a research project generating a process innovation. Simultaneously, they decide on the optimal degree of product differentiation by investing in another research project generating a product innovation. On the second stage, firms choose quantities. Applying backward induction, we first analyze firms’ quantity decision in the second stage.

3. Quantity decisions

The quantity game is a partial equilibrium model with heterogeneous products and Cournot competition. Since we consider a market in which there are two firms offering quantities \( x_i \) and \( x_j \), firm \( i \)'s profit function is given by

\[
\pi_i(x_i, x_j) = x_i P_i(x_i, x_j, \delta) - c_i x_i.
\]

Each firm maximizes its profit given the quantity chosen by the other firm, such that the equilibrium solution of the Cournot subgame is given by

\[
x^*_i = \frac{2(a - c_i) - \delta(a - c_j)}{4 - \delta^2}.
\]

(1)

The game obviously has a unique equilibrium in pure strategies in which the firms choose \( x^*_i(c_i, c_j, \delta) \) and \( x^*_j(c_j, c_i, \delta) \). For further analysis it is useful to analyze comparative static properties of the optimal quantities with respect to changes in marginal costs and in the degree of product substitutability. Differentiation of (1) with respect to \( c_i \) and \( c_j \) yields:

\[
x^*_{i,c_i} = -\frac{2}{4 - \delta^2} < 0,
\]

(2)

\[
x^*_{i,c_j} = \frac{\delta}{4 - \delta^2} > 0.
\]

(3)

Differentiating both first-order conditions with respect to the degree of product differentiation \( \delta \), substituting and solving gives:

\[
x^*_{i,\delta} = \frac{2x^*_i - \delta x^*_i}{\delta^2 - 4} \geq 0 \quad \text{for} \quad \frac{\delta}{2} x^*_i \geq x^*_j,
\]

(4)

which simplifies to \( x^*_{i,\delta} = -x^*_i/(2 + \delta) < 0 \) for \( x^*_i = x^*_j \). As expected in a Cournot setting, each firm’s optimal quantity decreases with an increase of its own marginal costs and increases with the marginal costs of its rival. A firm’s reaction to a change in the level of product differentiation is twofold. If firms offer the same quantities, an increase in product differentiation (which corresponds to a decrease in \( \delta \)) leads to a higher optimal quantity.

\[11\] Note that second-order conditions are satisfied.
If firms’ quantities are sufficiently different, the ‘large’ firm’s quantity increases while the other firm’s quantity decreases in $\delta$.

The reduced form profit function of firm $i$ is given by

$$\pi_i^* (c_i, c_j, \delta_i, \delta_j) = x_i^*(c_i, c_j, \delta_i, \delta_j)^2.$$  \hspace{1cm} (5)

In the next section, the first stage of the game is analyzed, in which firms choose the optimal level of marginal costs and product differentiation through R&D investment.

4. The innovation decisions under R&D competition

Anticipating the outcome of the stage-two game, firms choose optimal R&D projects. Possible R&D projects are targeted at process innovation and at product innovation. Through the former they choose marginal costs of production and through the latter they choose a degree of product differentiation. Firms’ strategies are $(c_v, \delta_v) \in \mathbb{R}^2$, with $c_v \in [0, c^0]$ and $\delta_v \in [0, \delta^0]$ with $v = i, j$.

The Nash-equilibrium strategies of both firms $(c_i, \delta_i)$ and $(c_j, \delta_j)$ are defined by the mutual best-response property. That is for firm $i$:

$$(c_i, \delta_i) \in \arg\max_{c_i, \delta_i} \left\{ \prod_i = \pi_i^*(c_i, c_j, \delta_i, \delta_j) - K(c_i) - G(\delta_i) \right\},$$

and firm $j$’s maximization problem is defined analogously. Solving (6) with respect to $c_i$, substituting (3) and rearranging leads to the following first-order condition:

$$x_i^* \left( \frac{4}{\delta^2 - 4} \right) = K'$$

Maximizing firm’s profit with respect to $\delta_i$, we restrict attention to symmetric equilibria. Solving (6), then substituting $x_i^*$, using $x_i^* = x_j^*$ and rearranging yields:

$$x_i^{*2} \left( \frac{-2}{\delta + 2} \right) = G'.$$

Since firms are identical at the outset, we find analogous conditions for firm $j$. If we additionally assume that the following condition holds:

$$(\pi_{i\|i})^2 \leq (\pi_{i\|i} - K'') (\pi_{j\|i} - G'')$$

the model satisfies all conditions for the existence of a unique symmetric equilibrium in pure strategies.$^{12}$

$^{12}$ Checking all relevant derivatives reveals that firms’ individual profits are not supermodular in the two investments. Therefore, also asymmetric equilibria might exist which can presumably be characterized by one firm investing more while the other is less active with respect to both kinds of innovations.
5. The innovation decisions under R&D cooperation

Now suppose that firms form a research joint venture in which they coordinate their R&D activities while they remain competitors in the second stage of the game. If firms coordinate R&D activities in the first stage, they choose R&D investment as to maximize their joint profit, \( \pi_i + \pi_j \). Concerning the organization of R&D, two cases can be considered: (a) firms coordinate their research strategies by joint profit maximization, but conduct research in two separate labs, or (b) they coordinate their research strategies and achieve efficiency gains by building-up only one research unit. The first case can be characterized as R&D-cartelization, the second as RJV-cartelization. We will begin with the analysis of the first case.

5.1. R&D-cartel: joint profit maximization without efficiency gains

Since the second stage is unaffected by any cooperative agreement, only optimal quantities are presented here. Provided that firms are committed to choose a common level of marginal costs and product differentiation, they will offer symmetric quantities in regime \( k \).\(^{15}\)

\[
x_k^* = \frac{a - c_k}{2 + \delta}.
\]

Turning to the first stage, now assume that firms coordinate their strategies as to maximize joint profit but do not achieve efficiency gains because they still run two decreasing-returns research technologies. This more unrealistic scenario is included into analysis because it allows us to isolate the strategic effects of cooperative R&D investment and the influence of efficiency gains. Firms’ joint profit is

\[
\prod_k = \pi_i^*(c_i, \delta_i, c_j, \delta_j) + \pi_j^*(c_i, \delta_i, c_j, \delta_j) - K(c_i) - K(c_j) - G(\delta_i) - G(\delta_j).
\]

Optimization with respect to \( c_i \) leads to the following implicit function which gives optimal investment into process innovation under R&D-cartelization, for \( c_i^* = c_j^* = c_k^* \):

\[
x_k^2 \left( \frac{-2}{\delta + 2} + 1 \right) = K' \quad (10)
\]

Maximizing joint profit with respect to the level of product differentiation \( \delta_i \) yields the implicit function for optimal investment into product innovation under R&D-cartelization, for \( \delta_i^* = \delta_j^* = \delta_k^* \):

\[
x_k^2 \left( \frac{-4}{\delta + 2} + 1 \right) = G' \quad (11)
\]

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13 This is a common assumption in the IO literature on research joint ventures, based upon the fact that in most countries these joint ventures have to be registered and then are monitored by the antitrust authorities.

14 See Kamien et al. (1992) for this terminology.

15 Salant and Shaffer (1998) show that instead of investing identical amounts on cost reduction and then competing as equally matched rivals, the firms can often earn strictly higher joint profits by investing at the first stage to induce monopoly or at least asymmetric duopoly at the second stage. Taking into account the case-by-case permissive attitude towards RJVs in Europe and the US we consider the symmetric arrangement to be more realistic.
Let us now examine the relationship between the first-order necessary conditions if firms choose their research strategies so as to maximize only their individual profits, expressions (7) and (8), and compare these to the conditions when they maximize combined profits, expressions (10) and (11). Note that the two problems are comparable in the sense that, e.g. in case of process innovation

$$\frac{\partial \prod_k}{\partial c_i} = \frac{\partial \prod_i}{\partial c_i} + \frac{\partial \prod_j}{\partial c_i},$$

where $\prod_j$ is the (negative) externality conferred by firm $i$’s cost reduction on the profit of its rival $j$. Analogously, also the (positive) externality induced through product innovation by firm $i$ on the profit of firm $j$ is added to the competitive advantage externality the firm’s R&D effort has on its own profit through increasing the amount of differentiation of its competitor. Those externalities, positive or negative, are ignored when each firm chooses its R&D expenditure so as to maximize its own profit. They are internalized when the firms coordinate their R&D strategies, what causes the individual maximization problems to be equivalent to the joint maximization problem that would be solved by a single director of the R&D-cartel.

To determine the effect of those strategic terms, the first-order conditions given by (10) and (11) can be written as

$$\pi_{i_i} - K' + \beta \pi_{j_i} = 0, \tag{12}$$

and

$$\pi_{j_i} - G' + \beta \pi_{j_j} = 0, \tag{13}$$

respectively, with $\beta = 1$. By applying comparative statics with respect to $\beta$, the effects of adding these strategic terms to firms’ first-order conditions of profit maximization (and thus, the effect of internalized externalities on firms’ investment incentives) can be analyzed. Taking all variables as functions of $\beta$, differentiation of (12) and (13) with respect to $\beta$ yields:

$$\left(\pi_{i_i} - K'' + \beta \pi_{j_i} \right)\, dc_i + \left(\pi_{i_i} \delta_i + \beta \pi_{j_i} \delta_i \right)\, d\delta_i + \pi_{j_i} \, d\beta = 0,$$

$$\left(\pi_{j_i} + \beta \pi_{j_j} \right)\, dc_i + \left(\pi_{i_i} \delta_i - G'' + \beta \pi_{j_j} \delta_i \right)\, d\delta_i + \pi_{j_j} \, d\beta = 0.$$

Cramer’s rule leads to:

$$\text{sign} \left( \frac{dc_i}{d\beta} \right) = \text{sign}\left( -\left(\pi_{i_i} \delta_i - G'' + \beta \pi_{j_j} \delta_i \right)\pi_{j_i} + \pi_{j_i} \left(\pi_{i_i} \delta_i + \beta \pi_{j_j} \delta_i \right) \right),$$

$$\text{sign} \left( \frac{d\delta_i}{d\beta} \right) = \text{sign}\left( -\left(\pi_{i_i} - K'' + \beta \pi_{j_i} \right)\pi_{j_j} + \pi_{j_j} \left(\pi_{i_i} + \beta \pi_{j_i} \right) \right).$$

Since the sign of the right-hand sides of both expressions is ambiguous, obviously the slopes of the marginal R&D-cost functions determine whether R&D investment is increased

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16 Note that Eqs. (10) and (11) imply Eqs. (7) and (8), respectively, when $\beta = 0$. 
or decreased. Decreased investment in process innovation, e.g. $c_i \beta > 0$, means that the right-hand side of the first expression has to be positive, which is true in case:

$$G'' > \frac{\beta(\pi_{j,i})\pi_{j,k} - \pi_{j,i,j} - \pi_{i,k,j} + \pi_{i,k} \pi_{j,k}}{\pi_{j,i}} := \hat{G}''.$$  

Cooperation increases investment in product innovation, e.g. $\delta_i \beta < 0$, whenever the right-hand side of the second expression is negative, or if

$$K'' > \frac{-\beta(\pi_{j,i})\pi_{j,i} - \pi_{j,i,j} - \pi_{i,k,j} + \pi_{i,k} \pi_{j,k}}{\pi_{j,i}} := \hat{K}''.$$  

These conditions on the R&D-cost functions can only be met, if they allow for the existence of an equilibrium. To ensure that there exists an equilibrium for all values of $\beta$, the second-order condition, given by (9), has to be transformed to:

$$G'' \geq \frac{\pi_{i,k} + \beta \pi_{j,i}}{\pi_{j,i}} = \frac{(\pi_{i,k} + \beta \pi_{j,i})^2}{\pi_{j,i}} - K''.$$  

Substituting $\hat{K}''$ into this new second-order condition of profit maximization, reveals that whenever $K'' < \hat{K}''$ it is necessary for the equilibrium to exist that $G'' \geq \hat{G}''$. Similarly, $G'' < \hat{G}''$ requires that $K'' \geq \hat{K}''$. This leads to the following proposition.

**Proposition 1.** A coordination of strategies induces firms to invest:

(i) less in process innovation and less in product differentiation compared to the non-cooperative equilibrium if $K'' < \hat{K}''$ or

(ii) more in process innovation and more in product innovation if $G'' < \hat{G}''$,

(iii) or more in product differentiation and less in process innovation if $K'' > \hat{K}''$ and $G'' > \hat{G}''$.

**Proof.** See the arguments described earlier. □

Hence, it can definitely be excluded that firms invest more in process innovation, $c_i \beta < 0$, and less into product differentiation, $\delta_i > 0$, when they coordinate their research strategies in an R&D-cartel as compared to the competitive equilibrium. Keeping in mind that $\hat{K}''$ and $\hat{G}''$ are functions of R&D investment expenditures one can simplify the interpretation as follows: if both marginal R&D-cost functions are sufficiently steep (such that $K'' > \hat{K}''$ as well as $G'' > \hat{G}''$) coordination of strategies induces firms to invest more in product differentiation, but less in process innovation. This finding corresponds to the general economic insight, that the internalization of positive externalities should increase incentives to conduct R&D while the opposite should hold for negative externalities. On the other hand in case marginal R&D cost increase slowly for process innovation, that is $K'' < \hat{K}''$ firms will invest less in both kinds of innovation compared to the non-cooperative equilibrium. If costs for process innovation are low, negative externalities become strong.

17 See De Bondt and Veugelers (1991) for a comprehensive discussion of the effects of investment externalities.
for competing firms and the internalization of those strong negative externalities overcompensates positive externalities. While if the slope of marginal R&D cost is low for product innovation, that is \( G'' < \hat{G}'' \), firms invest more in both kinds of innovation. If firms can easily differentiate their products, the internalization of positive externalities overcompensates negative externalities.

5.2. RJV-cartel: joint profit maximization with efficiency gains

A change of assumptions concerning the organization of R&D affects again only the first stage of the game. Since all analytical steps are corresponding to those discussed previously, formalities will be confined to the second-stage first-order conditions of joint profit maximization and the maximization problem itself, which is given by

\[
\prod_v = \pi_v^i(c_i, \delta_i, c_j, \delta_j) + \pi_v^j(c_i, \delta_i, c_j, \delta_j) - \frac{1}{2}K(c_i) - \frac{1}{2}K(c_j) - \frac{1}{2}G(\delta_i) - \frac{1}{2}G(\delta_j).
\]

Optimal investment into process innovation under RJV-cartelization, for \( c^*_i = c^*_j = c^*_s \) is determined by

\[
\lambda^*_s \left( \frac{-4}{2 + \delta} \right) = K'. \tag{14}
\]

For optimal product innovation \( \delta^*_i = \delta^*_j = \delta^*_s \) under RJV-cartelization, the first-order condition is

\[
\lambda^*_2 \left( \frac{-8}{2 + \delta} \right) = G'. \tag{15}
\]

The optimization problems under the two regimes, R&D competition and RJV-cartelization, now differ in two aspects: first, under cooperation firms consider the effects of their investment on their rival’s profit, and second, R&D-cost functions change due to effort sharing. A comparison of optimal investment under the two regimes leads to the following proposition.

**Proposition 2.** If firms form an RJV-cartel this leads to lower marginal costs and more differentiated products than R&D competition. Under RJV-cartelization firms proportionally invest more in product innovation than under competition.

**Proof.** For a comparison of firms’ optimal R&D investment under the two regimes we characterize the resulting equilibrium points in the \((c, \delta)\) space: each pair of first-order conditions gives us an implicit function \( c(\cdot) = c(\delta) \). Comparing (7) and (14) shows, that for \( \hat{\delta} = \delta = 0 \) we find \( c_s < c_i \), since \(-2x^*_s(\cdot, 0) < -x^*_i(\cdot, 0)\) and \( K'' < 0 \) and \( K'' > 0 \) for \( v = s, i \) and \( \hat{\delta} \) denoting the level of product differentiation under cooperation. Furthermore, \( c_i = c_s \) for \( \delta = 1 \), since \(-4/(2 + \delta) = 4/(\delta^2 - 4)\) if \( \delta = 1 \). Now assume (for contradiction) that there exists some \( \hat{\delta} = \delta < 1 \) such that \( c_i = c_s \). This implies that for \( \hat{\delta} = \delta \) and \( c_i = c_s \) also \( x^*_i(c_i, \delta) = x^*_s(\cdot, \delta) \) and \( K'(c_i) = K'(c_s) \) holds, and therefore, \(-4/(2 + \delta) = 4/(\delta^2 - 4)\) has to hold as well, which is true for \( \hat{\delta} = \delta = 1 \), and thus, leads to a contradiction. Hence, since \(-4/(2 + \delta) \leq 4/(\delta^2 - 4)\) if \( \delta \leq 1 \) both functions can lead
Fig. 1. Implicit functions of first-order conditions.

to the same \( c_i = c_s \) only for \( \hat{\delta} = \delta = 1 \). Applying the implicit function theorem, the slopes of the functions \( c_{i\delta} \) and \( c_{s\delta} \) can be determined.\(^{18}\) Since \( c_{s\delta} > 0 \) and \( c_{i\delta} \geq 0 \) for \( \delta \leq 2/3 \) (and \( c_{i\delta} \leq 0 \) for \( \delta \geq 2/3 \)) in the \( (c, \delta) \) space the function \( c_s(\delta) \) lies always underneath the function \( c_i(\delta) \). Therefore, under RJV-cartelization firms choose for each \( \delta \) a lower level of marginal costs \( c \).

Now consider (8) for the competitive case and (15) for the cooperative case, respectively. For any given \( c_i = c_s \), we find \( \hat{\delta} < \delta \), with \( \delta = 2\delta_i \) and \( \hat{\delta} = 2\delta_s \), since for any given \( c \) we have \( -8/\delta + 2 < -2/(\delta + 2) \) and \( G' < 0 \) and \( G'' > 0 \). Applying again the implicit function theorem, from (8) and (15), we find that \( (\delta_{ci})^{-1} > 0 \) and \( (\delta_{cs})^{-1} > 0 \) (see Appendix A). Therefore, under cooperation firms choose for any given \( c \) a lower level of \( \delta \). In Fig. 1, all four functions are sketched.

A comprehension of arguments allows to conclude that under RJV-cartelization leads to lower marginal costs and to more product differentiation. Still, it is worthwhile to note that firms also change the proportion of optimal R&D investment as compared to R&D competition. Consider the ratios given by

\[
\frac{K_{c_i}}{G_{\delta_i}} = \frac{2}{(2-\delta)x_i^*},
\]

in the competitive case, and

\[
\frac{K_{c_s}}{G_{\delta_s}} = \frac{1}{2x_s^*},
\]

under cooperation. Due to the fact that \( c_s < c_i \) and \( \hat{\delta} < \delta \) for equilibrium quantities the following holds: \( x_i^* > x_s^* \), since \( x_{\delta_i}^* < 0 \) and \( x_{\delta_s}^* < 0 \). Hence, \( K_{c_i}/G_{\delta_i} > K_{c_s}/G_{\delta_s} \), meaning

\(^{18}\) See Appendix A.
that under competition firms proportionally invest more in process innovation than under cooperation.

The analysis of the impact of cooperative decision making on firms’ innovation strategies indicates that, under ‘preferences for product variety’, it seems most likely that investment is shifted towards product innovation.

An RJV-cartel avoids wasteful duplication of R&D efforts, and thus, yields lower marginal costs and more differentiated products than R&D competition. Cost sharing together with the internalization of positive externalities overcompensate the effect of internalization of negative externalities.

On the other hand, the R&D-cartel, relying only on the beneficial effects of strategy coordination, is more sensitive towards the relative efficiency of R&D investment: the slope of marginal R&D cost low for product innovation, firms invest more in both kinds of innovation and it is easily checked that investment is shifted towards product innovation. The reason is that if firms can easily differentiate their products, positive externalities are strong. The pure internalization of those positive externalities is sufficient to overcompensate the effect from internalizing negative externalities from process innovation. In case both marginal R&D-cost functions are sufficiently steep, a coordination of strategies induces firms to invest more in product differentiation, but at the same time less in process innovation.

To summarize we can point out that strategic considerations do have an impact on firms’ R&D portfolio. The preceding analysis allows us to conjecture that in countries in which antitrust authorities treat R&D cooperation between competitors more favorable, we should observe proportionately more product innovation.

6. Changes in market size

Up to this point, market size was considered to be constant. If consumers’ prohibitive price \( a \) is interpreted as the market potential, comparative static properties of firms’ optimal investment rules with respect to \( a \) can indicate the effects of changes in the market size. To be more precise, the symmetric equilibrium quantity in case of R&D competition is given by \( x_i^* = x_i^* (a, c_i^*(a), c_j^*(a), \delta_i(a) + \delta_j(a)) \). Hence, it is straightforward to analyze comparative static properties. If \( a \) changes, the first-order conditions have to remain satisfied:\(^{19}\)

\[
\begin{align*}
c_i(a) (\pi_i c_i - K''') + \delta_i(a) \pi_i c_i b_i + \pi_i c_i a &= 0, \\
\delta_i(a) (\pi_i b_i - G''') + c_i(a) \pi_i c_i b_i + \pi_i b_i a &= 0.
\end{align*}
\]

Cramer’s rule implies that the following relations hold:

\[
\begin{align*}
\text{sign} \left( \frac{dc_i}{da} \right) &= \text{sign} \left( - (\pi_i c_i b_i - G''') \pi_i c_i a + \pi_i c_i b_i \pi_i b_i a \right), \\
\text{sign} \left( \frac{d\delta_i}{da} \right) &= \text{sign} \left( - (\pi_i b_i - K''') \pi_i b_i a + \pi_i c_i a \pi_i b_i c_i \right).
\end{align*}
\]

\(^{19}\) Note that \( \pi_i c_i < 0 \) and \( \pi_i b_i < 0 \).
Obviously, as long as $\pi_{ci} \delta_i \geq 0$, which is the case if $\delta \leq 2/3$, we find $\delta_i < 0$ and $c_{ia} < 0$.

R&D investment in both kinds of innovation is increased when the market size increases.

In case $\pi_{ci} \delta_i < 0$, the signs of both expressions depend on the slopes of the marginal R&D-cost functions. Assume first, firms would increase their investment in product differentiation, which means that $\delta_i < 0$. For this to be true, expression (18) has to have a negative sign, which is the case if

$$K'' > \frac{\pi_{ci} \delta_i x_{ia} + c_{ia} x_{ici}}{\pi_{ci} a} \quad := \tilde{K}'' .$$

Now assume that investment in process innovation would be increased as well, e.g. $c_{ia} < 0$. This would be the case if

$$G'' > \frac{\pi_{ci} \delta_i x_{ia} + c_{ia} x_{ici}}{\pi_{ci} a} \quad := \tilde{G}'' .$$

Substituting $\tilde{K}''$ into the second-order condition of profit maximization, given by (9), shows that whenever $K'' < \tilde{K}''$ the existence of a symmetric pure strategy equilibrium requires $G'' \geq \tilde{G}''$. Similarly, $G'' < \tilde{G}''$ requires $K'' \geq \tilde{K}''$. This leads to the next proposition.

**Proposition 3.** As market size increases, firms invest:

(i) *more in both kinds of innovation and proportionally invest more in product innovation* if $\pi_{ci} \delta_i \geq 0$, or if $\pi_{ci} \delta_i < 0$ and $K'' > \tilde{K}''$ as well as $G'' > \tilde{G}''$, or

(ii) *more in product differentiation and less in process innovation* if $\pi_{ci} \delta_i < 0$ and $K'' < \tilde{K}''$.

(iii) *more in process innovation and less in product differentiation* if $\pi_{ci} \delta_i < 0$ and $G'' \geq \tilde{G}''$.

**Proof.** For the sings of $\delta_i$ and $c_{ia}$ see the earlier arguments. Next, consider (16). Differentiation respect to $a$ yields:

$$\frac{\partial K_{ci} / G_{bi}}{\partial a} = \frac{-2(\delta_i x_{ia} + (2 - \delta)(x_{ia}^a + \delta_i x_{ia} + c_{ia} x_{ici}))}{(-\delta_i x_{ia}^a + (2 - \delta)(x_{ia}^a + \delta_i x_{ia} + c_{ia} x_{ici})^2)} < 0 .$$

Taking into account that $x_{ia}^a > 0$ and that $x_{ia}^a < 0$ as well as $x_{ia}^a < 0$, it is easily checked that the ratio on the right-hand side is negative whenever $\delta_i < 0$ and $c_{ia} < 0$. Thus, firms proportionately invest more in product innovation the larger the market. \hfill \square

Hence, it can definitely be excluded that under ‘preferences for product variety’ an increase in consumers’ willingness-to-pay leads to less process innovation, $c_{ia} > 0$, as well as less product innovation, $\delta_i > 0$. To the contrary, given that both marginal cost functions are sufficiently steep, or products are sufficiently differentiated at the outset, an increase in the market size will induce firms to increase their R&D investment and to shift their investment towards product innovation.\(^{20}\) If marginal R&D cost of product innovation

\(^{20}\) Of course, we have again to keep in mind that $\tilde{K}''$ as well as $\tilde{G}''$ are functions of R&D investment expenditures.
increase slowly, that is $G'' < \tilde{G}''$, investment in product innovation will still be increased, but process innovation will be decreased. Analogously, in case the slope of marginal R&D cost is low for process innovation, that is $K'' < \tilde{K}''$, this leads to more process innovation and at the same time to less product innovation. It can further be shown that firms increase their R&D investment when market size increases if they coordinate their R&D decisions, independently from the specific form of agreement (see Appendix B).

In order to get some intuition for these results it is helpful to think in terms of strategic effects. Any investment into a reduction of marginal production costs by one firm leads to a lower profit for the other firm (negative externality). In this model, a process innovation by one firm also decreases the other firm’s marginal profit of its own process innovation. Therefore, process innovations are ‘strategic substitutes’ since a more aggressive move by firm $i$ (in the sense of increased investment in process innovation, and thus, lower marginal production costs) leads to a less aggressive move by firm $j$ (with respect to both kinds of innovations). Product innovations impose a positive externality on the other firm’s total as well as marginal profit and thus are ‘strategic complements’ since a more aggressive move by firm $i$ induces firm $j$ to invest more into product innovation. The effect on process innovation depends on the degree of product substitutability: if products are sufficiently differentiated, $\delta \leq 2/3$, product innovation by one firm is a ‘strategic substitute’ for process innovation by the other firm, for more homogenous products firm $i$’s product innovation is a ‘strategic substitute’ to firm $j$’s process innovation.

An increase in the market size has a direct positive effect on marginal returns of investment for both kinds of innovation. While the substitutive effect of firms’ process innovations remains unchanged, all other strategic effects are reinforced in a larger market. If products are sufficiently differentiated, $\delta \leq 2/3$, investment in both kinds of innovations is unambiguously increased in a larger market. The complementary effect of firm $i$’s product innovation on firm $j$’s product innovation overcompensates the substitutive effect of its process innovation on $\delta_j$, while the substitutive effect of $c_j$ on $c_i$ overcompensates the complementary effect of $\delta_j$ on $c_i$. Thus, investment is shifted towards product innovation.

If products are less differentiated, $\delta > 2/3$, product innovation $\delta_j$ becomes a strategic substitute for process innovation $c_i$, and therefore, more product innovation by firm $j$ leads to less process innovation by firm $i$. This now adds to the substitutive effect of $c_j$ on $c_i$, (i) If changes in $c$ and $\delta$ are both sufficiently expensive ($K'' > \tilde{K}''$ as well as $G'' > \tilde{G}'')$ this effect is not strong enough to overcompensate the positive direct effect of an increased market size. We observe an increase in R&D investment and proportionately more investment into product innovation.

If products are less differentiated, $\delta > 2/3$. (ii) Changes in $\delta$ are larger than changes in $c$ (since $G'' < \tilde{G}''$ and $K'' > \tilde{K}'$), strategic effects of firms’ product innovations are reinforced. The strong substitutive effect of $\delta_j$ on $c_i$ which again adds to the substitutive effect of $c_j$ on $c_i$ now overcompensates the positive direct effect of an increased market size on firm $j$’s marginal profit of process innovation. The complementary effect of $\delta_j$ on $\delta_i$ is reinforced and overcompensates the substitutive effect of process innovation $c_j$ on $\delta_i$. We observe less (total) investment in cost reduction and more in product differentiation. (iii)

21 Note, that $\pi_{i\delta_j} < 0$ as well as $\pi_{i\delta_i} < 0$, while $\pi_{i\delta_j} > 0$ and $\pi_{i\delta_i} \geq 0$ for $\delta \leq 2/3$. See Bulow et al. (1985) for a detailed discussion of strategic substitutes and strategic complements.
If changes in \( c \) are larger than changes in \( \delta \) (since \( G'' > \tilde{G}'' \) and \( K'' < \tilde{K}'' \)), the weaker complementary effect of \( \delta_j \) on \( \delta_i \) and the positive effect of an increased market size are both overcompensated by the strong substitutive effect from \( c_j \) on \( \delta_i \). The substitutive effects of \( c_j \) on \( c_i \) (which does not change with market size) and of \( \delta_j \) on \( c_i \) are not strong enough to overcompensate the positive direct effect of a larger market size on marginal profits of process innovation. We observe less (total) product innovation and more process innovation.

Obviously, market size also has a strong effect on firms’ innovative activities. The findings in this section support Abernathy and Utterback’s (1982) thesis in the sense that in an early stage of a product life-cycle, when market potential is large and production processes are not yet optimized for mass production, firms invest more into product innovation. Later in the life-cycle, when market potential decreases and products are sufficiently differentiated, investments is shifted towards process innovation.

7. Welfare

When analyzing R&D cooperation in this setting one may also consider welfare implications. Welfare is given by

\[
W = 2(a - c_i) x_i - (1 + 2 \hat{\delta}) x_i^2 - bK(c_i) - bG(\delta_i)
\]

with \( b = 1 \) and \( l = s \) for R&D cooperation in the sense of RJV-cartelization, and \( b = 2 \) and \( l = i \) for competition. First note that here the firms always have an incentive to cooperate in their R&D activities. Given \( \prod_s \), firm \( i \) has chosen \( c_s^* \neq c_i^* \) and \( \delta_s^* \neq \delta_i^* \) as the maximizing arguments although \( c_i^* \) and \( \delta_i^* \) were also feasible solutions. Hence, if firm \( i \) decides to cooperate it must be the case that the following holds:

\[
\pi(c_i^*, \delta_i^*) - \frac{1}{2}(K(c_i^*) + G(\delta_i^*)) \geq \pi(c_s^*, \delta_s^*) - (K(c_s^*) + G(\delta_s^*)).
\]

Proposition. If firms have an incentive to form an RJV-cartel, this cooperation increases welfare.

Proof. Note that (19) can be rewritten as

\[
(a - c_i^*) x_i^* - \frac{1}{2}(1 + \hat{\delta}) x_i^2 - \frac{1}{2}(K(c_i^*) + G(\delta_i^*)) \geq (a - c_s^*) x_s^* - \frac{1}{2}(1 + \delta) x_s^2 - (K(c_s^*) + G(\delta_s^*)).
\]

with \( \delta = 2 \hat{\delta} \) for the competitive case and \( \hat{\delta} = 2 \delta_i^* \) under cooperation. Whereas welfare is increased whenever

\[
(a - c_s^*) x_s^* - \frac{1}{2}(1 + \hat{\delta}) x_s^2 - \frac{1}{2}(K(c_s^*) + G(\delta_s^*)) \geq (a - c_i^*) x_i^* - \frac{1}{2}(1 + \delta) x_i^2 - (K(c_i^*) + G(\delta_i^*)).
\]

Given that (19) is satisfied, this can be rewritten as

\[
\frac{1}{2}(1 + \hat{\delta}) x_s^2 - \frac{1}{2}(1 + \delta) x_s^2 \geq 0 \Leftrightarrow (1 + \hat{\delta}) x_s^2 \geq (1 + \delta) x_s^2.
\]
Substituting $x^*_i$ and $x^*_s$ and rearranging leads to:

$$\frac{(a - c^*_i)^2}{(a - c^*_s)^2} \geq \frac{(1 + \delta)(2 + \hat{\delta})^2}{(1 + \hat{\delta})(2 + \delta)^2}.$$  \hspace{1cm} (21)

This inequality holds, because due to $c^*_i > c^*_s$ the right-hand side of (21) is larger than one and due to $\hat{\delta} < \delta$ the left-hand side is smaller than one. Obviously, if (19) is satisfied, cooperation leads to more investment into both kinds of R&D, also welfare will be increased.

It is easily seen that welfare is increasing in a reduction of $(c, \delta)$ as long as the increase in R&D cost does not overcompensate the gains of producer and consumer surplus or, in other words, as long as firms have an incentive to cooperate. This is not surprising since consumers as well as producers benefit from an increase in product differentiation and from reducing marginal costs. The only limiting factor is the increase of R&D cost.

If firms form an R&D cartel (without efficiency gains) when they cooperate, welfare implications are ambiguous. If marginal R&D-cost functions are sufficiently steep, cooperative firms differentiate their products more which benefits the consumers. But marginal costs are higher under this kind of cooperation than under competition. Therefore, the effect on consumer surplus depends strongly on the relative strength of the two effects. Without assuming concrete functions for R&D costs no ‘general’ conclusion can be drawn. But when marginal costs for product innovation do not increase too rapidly, also cooperation in the sense of R&D-cartelization increases welfare. The positive competitive spillover on the product market can (under the described conditions concerning the R&D-cost functions) outweigh negative externalities from process innovation. It can therefore take the role of technological spillovers and cause welfare improvements even in case of cooperative decision making without cost sharing. These findings qualify previous findings on welfare effects of cooperative R&D, i.e. of D’Aspremont and Jacquemin (1988), and allows us to call for a more favorable treatment of research joint ventures than implied by solely taking into account technological spillovers. If cooperative agreements on R&D include horizontal product innovations in an existing product line they tend to be socially beneficial.

8. Conclusion

In this paper, we have used a simple structure of monopolistic competition which enables us to analyze firms’ R&D decision in a differentiated industry if consumers have preferences for product variety, and when firms can determine marginal costs and product substitutability simultaneously. In particular, we compare firms’ R&D decision under competition and cooperation and conduct some comparative statics concerning the market size, in order to suggest further explanations for firms’ different R&D portfolios.

We find that if firms coordinate their R&D activities in an RJV-cartel, they will reduce marginal costs and product substitutability more than under competition due to efficiency gains. They will also proportionally invest more into product innovation since the strategic importance of marginal costs diminishes if firms cooperate (internalization of negative
The cost sharing agreement together with the internalization of positive competitive spillovers outweigh the incentive to reduce investment in process innovations. In this case of R&D cooperation, it is clear that the welfare implications of joint research are positive since also consumers benefit from lower costs and more differentiated products.

If firms cooperate, but for any reason cannot organize R&D in one lab to increase R&D efficiency, they still proportionally invest more into product innovation. However, in this case, they differentiate their products more but reduce marginal costs less than under competition. The welfare implications of an RJV in which firms only eliminate strategic components but do not reduce overall R&D costs are therefore ambiguous. Only if product differentiation is not too expensive, firms increase their investment for both kinds of innovation, and welfare increases. In this scenario, positive competitive spillovers from the product market can play a similar role as technological spillovers on the R&D market. If they are sufficiently strong, any cooperative agreement is welfare enhancing. These findings are in a sense complementary to Esran and Gallini (1996). While they demonstrate the impact of patent policy on firms’s product and process innovation decisions, we claim that also antitrust policy towards R&D cooperation has an influence on firms’ innovative decisions, and thus, plays a role in directing technological change.

Furthermore we find that the size of a market plays an important role for the relative strength of different externalities and thus may in fact help to explain different innovation strategies. The larger is the market size (in the sense of market potential) the more firms invest in R&D and the more they shift their investment towards product innovation, provided that R&D efficiencies of both kinds of innovation are rather similar or products are sufficiently differentiated at the outset. Our analysis supports Abernathy and Utterback (1982) thesis in the sense that in an early stage of a product life-cycle, when market potential is large and production processes are not yet optimized for mass production, firms invest more into product innovation. Later in the life-cycle, when market potential decreases and products are sufficiently differentiated, investment is shifted towards process innovation.

Due to the problem that there is not a common understanding of how to depict product innovations, one could of course think of different demand structures generating vertical or Hotelling’s horizontal product differentiation. The results derived will not naturally generalize to other assumptions concerning demand structure. Still, it is interesting to note that they are in line with the results of Klepper (1996) who analyzes the non-strategic patterns of process and product innovation over the product life-cycle under more general assumptions on demand as well as with the thesis of Abernathy and Utterback (1982).

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Appendix A

For any implicit function $c_\nu = c_\nu(\delta_\nu)$ given by the function $F(c_\nu(\delta_\nu), \delta_\nu) = 0$, we find $c_{\nu \delta_\nu} = -F'_\delta / F'_{c_\nu}$. The functions $c_{i\delta_i}$ and $\delta_{i\nu}$ can be derived from (7) and (8), respectively as

$$c_{i\delta_i} = -\frac{x^*_i 4(2 - 3\delta)/(\delta^2 - 4)^2}{\prod_{i \neq i} c_{i\delta_i}}$$

and

$$\delta_{i\nu} = -\frac{x^*_i \delta/(\delta + 2)^2}{\prod_{i \neq i} \delta_{i\nu}}$$

The denominators of both functions are negative since the second-order conditions for profit maximization are assumed to be satisfied. Therefore, the signs of $c_{i\delta_i}$ and $\delta_{i\nu}$ are given by the signs of the numerators, which means that $\delta_{i\nu}$ is positive and $c_{i\delta_i} < 0$ for $\delta > 2/3$. Using $\partial c_\nu(2\delta_\nu)/\partial \delta_\nu = 2c_\nu$ with $\nu = i, s$, shows that $\delta_{i\nu} > 0$ and $c_{i\delta_i} > 0$ for $\delta < 2/3$.

For the cooperative case, $c_s$ can be derived from (10):

$$c_s = -\frac{x^*_s 8/(\delta + 2)^2}{\prod_{s \neq s} c_{s}} > 0$$

Again, the sign of $c_s$ is given by the sign of the numerator, which is positive. Applying again the implicit function theorem, from (11) we find that

$$\delta_{s\nu} = -\frac{x^*_s 16/(\delta + 2)^2}{\prod_{s \neq s} c_{s}} > 0$$

holds, since the numerator is again negative and the denominator is positive. Therefore, also $\delta_{s\nu} < 0$. Inverting $\delta_{s\nu}$ and $c_{i\delta_i}$ allows us to depict all functions in one graphic.

Appendix B

Under cooperation firms invest more into both kinds of innovation since applying Cramer’s rule yields for process innovations, with $\nu = s, k$:

$$\text{sign} \left( \frac{d c_\nu}{d a} \right) = \text{sign} \left( -\prod_{c_{i\delta_i} < 0} c_{i\delta_i} + \prod_{\delta_{s\nu} < 0} \delta_{s\nu} \right) \Rightarrow \frac{d c_\nu}{d a} < 0.$$  

and for product innovations it yields:

$$\text{sign} \left( \frac{d \delta_\nu}{d a} \right) = \text{sign} \left( -\prod_{c_{i\delta_i} < 0} c_{i\delta_i} + \prod_{\delta_{s\nu} < 0} \delta_{s\nu} \right) \Rightarrow \frac{d \delta_\nu}{d a} < 0.$$  

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